The response of a linear single-degree-of-freedom oscillator to periodic excitation

Task: determine the steady-state response of the oscillator to the given oscillation of the base. Clue: the kinematics of the base can be decomposed into a Forurier series.

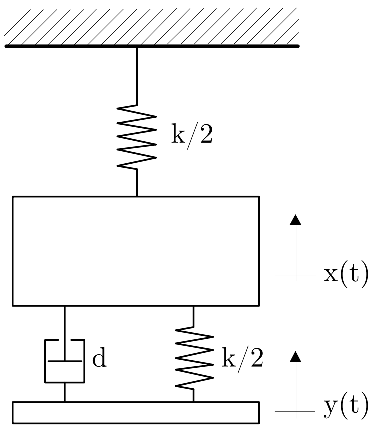

Figure 1: Oscillator.

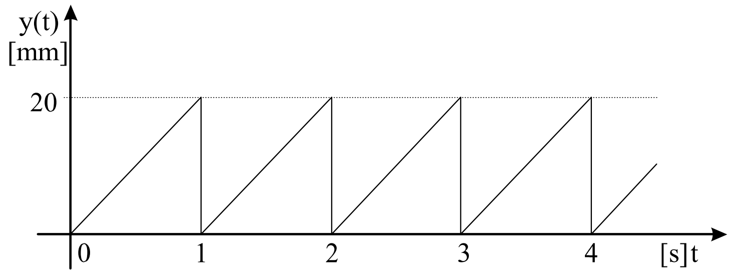

Figure 2: Base kinematics.

Oscillator data:

$m = 2$ kg	mass
$k = 1800\ \frac{\mathrm{N}}{\mathrm{m}}$	spring constant
$\delta = 0.1$	damping ratio
$\omega_0 = \sqrt{\frac{k}{m}}$	$\omega_0 = 30$ rad/s resonance frequency
$d = 2\delta\omega_0 m$	$d = 12\ \frac{\mathrm{Ns}}{\mathrm{m}}$ damping coefficient
$a = 20$ mm	excitation amplitude
$t = 1$ s	excitation period

The equation of motion for the oscillator in Fig. 1 is:

$$m\ddot{x} + d\dot{x} + kx = \frac{1}{2}ky + d\dot{y}. \tag{1}$$

Excitation kinematics of the base, shown in Fig. 2 can be written in a Fourier series:

$$y(t, N) = \frac{a}{2} - \frac{a}{\pi}\sum_{n=1}^{N}\frac{1}{n}\sin(2\pi n t). \tag{2}$$

The approximate kinematics of the base with respect to the number of considered terms in equation (2) is depicted in Fig. 3.

where:
t time [s]
N number of considered terms of the series

The particular solution of the equation of motion (1) can be calculated by substituting the base kinematics with its Fourier series:

$$X_{\mathrm{p}}(t, N) = \frac{a}{4}\sum_{n=1}^{N}[C_1(n)\sin(\omega(n)t) + C_2(n)\cos(\omega(n)t)] + \sum_{n=1}^{N}[D_1(n)\sin(\omega(n)t) + D_2(n)\cos(\omega(n)t)], \tag{3}$$

where:

$$\omega(n) = 2\,\pi\,n$$

$$Ampl\_K(n) = -\frac{1}{2}\omega_0^2\frac{a}{n\,\pi}$$

$$Ampl\_D(n) = -\frac{2\,a\,d}{m}$$

$$C_1(n) = \frac{Ampl\_K(n)\,\left(\omega_0^2 - \omega^2(n)\right)}{\left(\omega_0^2 - \omega^2(n)\right)^2 + (2\,\delta\,\omega_0\,\omega(n))^2}$$

$$C_2(n) = \frac{Ampl\_K(n)\cdot 2\,\delta\,\omega_0\,\omega(n)}{\left(\omega_0^2 - \omega^2(n)\right)^2 + (2\,\delta\,\omega_0\,\omega(n))^2}$$

$$D_1(n) = \frac{Ampl\_D(n)\cdot 2\,\delta\,\omega_0\,\omega(n)}{\left(\omega_0^2 - \omega^2(n)\right)^2 + (2\,\delta\,\omega_0\,\omega(n))^2}$$

$$D_2(n) = \frac{Ampl\_D(n)\,\left(\omega_0^2 - \omega^2(n)\right)}{\left(\omega_0^2 - \omega^2(n)\right)^2 + (2\,\delta\,\omega_0\,\omega(n))^2}$$

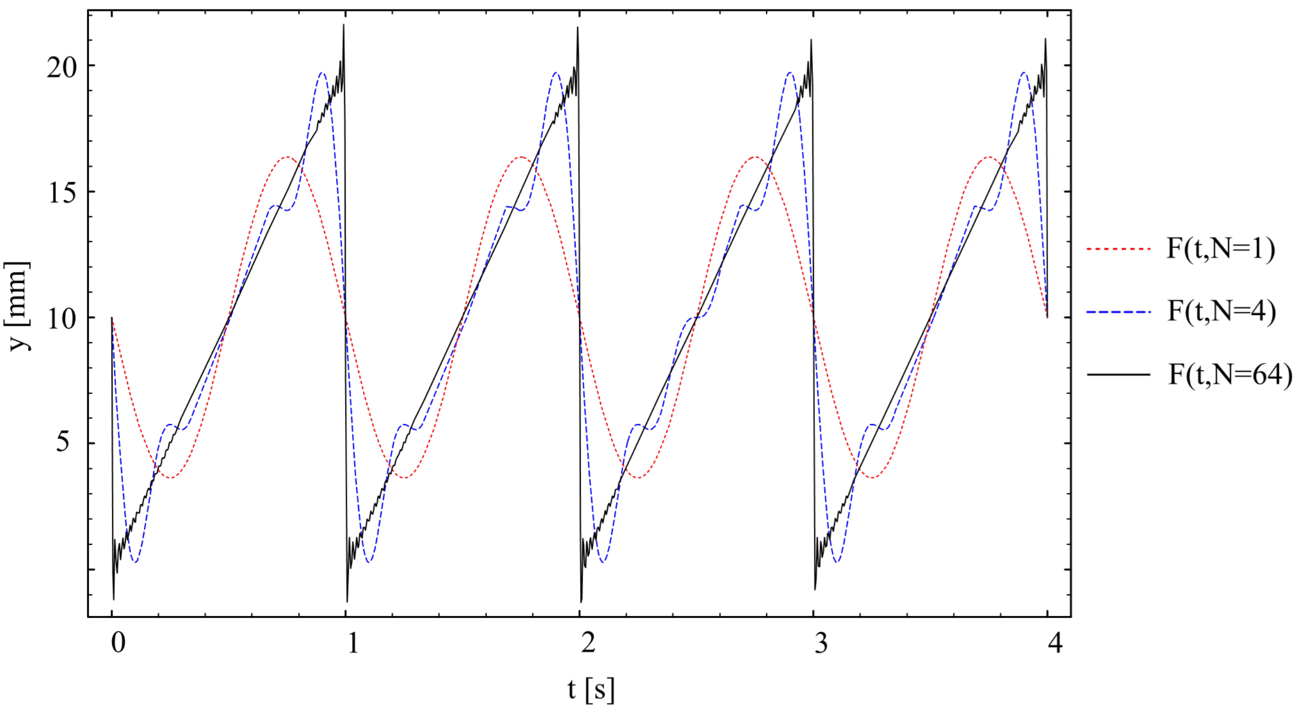

Figure 3: Base kinematics approximated by the Fourier series.

The effect of the number of considered terms to the particular solution is shown in Figures 4 and 5.

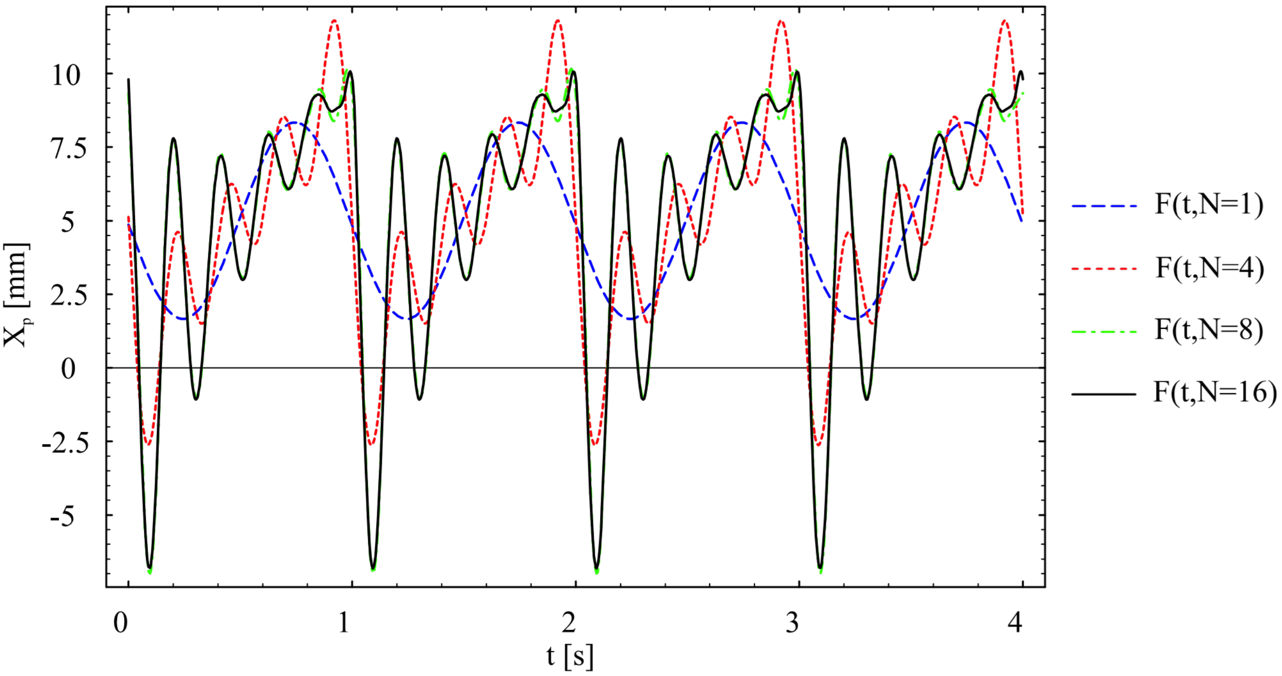

Figure 4: Oscillator response.

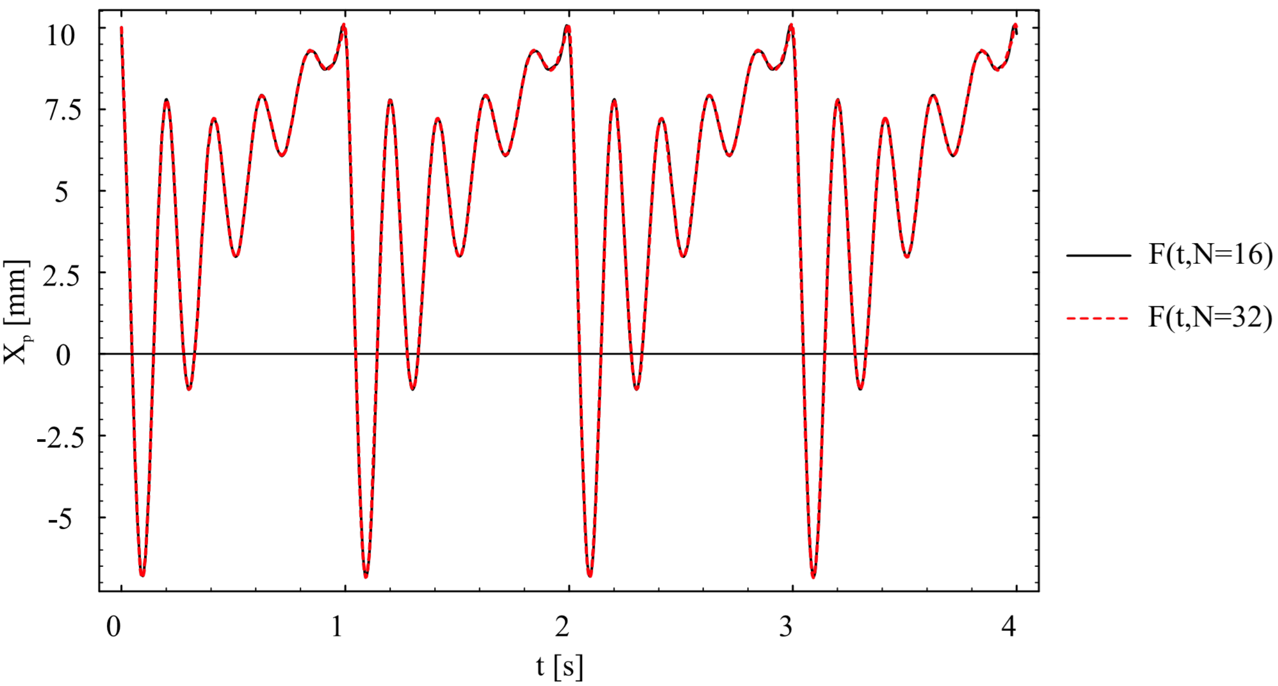

Figure 5: Oscillator response.