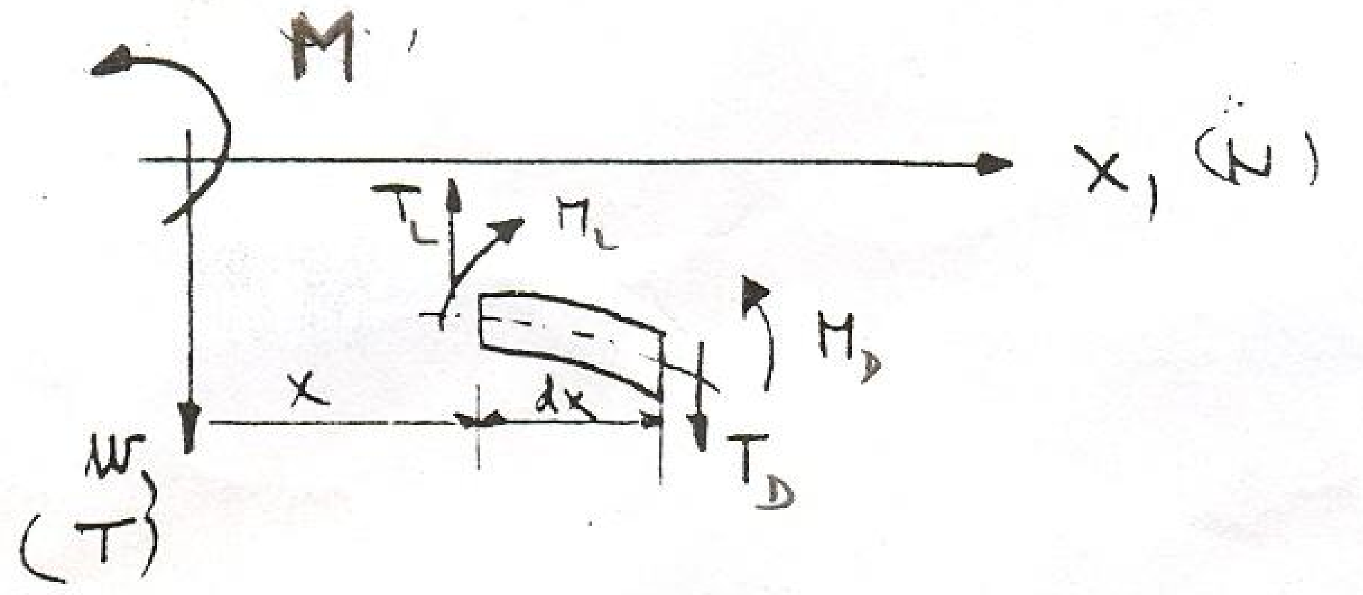

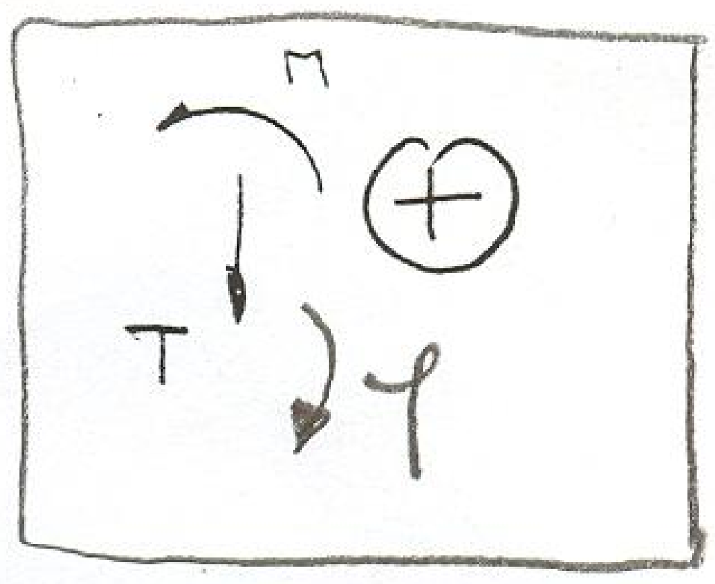

Boundary condition	At left end ($x = 0$)	At right end ($x = l$)
Free end (bending moment = 0, shear force = 0)	$x = 0$ $-EI\frac{\partial^2 w}{\partial x^2}(0,t) = 0$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = 0$	$+EI\frac{\partial^2 w}{\partial x^2}(l,t) = 0$ $x = l$ $+\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = 0$
Fixed end (deflection = 0, slope = 0)	$x = 0$ $w(0,t) = 0$ $\frac{\partial w}{\partial x}(0,t) = 0$	$x = l$ $w(l,t) = 0$ $\frac{\partial w}{\partial x}(l,t) = 0$
Simply supported end (deflection = 0, bending moment = 0)	$x = 0$ $w(0,t) = 0$ $-EI\frac{\partial^2 w}{\partial x^2}(0,t) = 0$	$x = l$ $w(l,t) = 0$ $+EI\frac{\partial^2 w}{\partial x^2}(l,t) = 0$
Sliding end (slope = 0, shear force = 0)	$x = 0$ $\frac{\partial w}{\partial x}(0,t) = 0$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = 0$	$x = l$ $\frac{\partial w}{\partial x}(l,t) = 0$ $+\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = 0$
End spring (spring constant = k)	$x = 0$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = +k\,w(0,t)$ $-EI\frac{\partial^2 w}{\partial x^2}(0,t) = 0$	$x = l$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = -k\,w(l,t)$ $EI\frac{\partial^2 w}{\partial x^2}(l,t) = 0$
End damper (damping constant = c)	$x = 0$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = +c\frac{\partial w}{\partial t}(0,t)$ $-EI\frac{\partial^2 w}{\partial x^2}(0,t) = 0$	$x = l$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = -c\frac{\partial w}{\partial t}(l,t)$ $EI\frac{\partial^2 w}{\partial x^2}(l,t) = 0$
End mass (mass = m with negligible moment of inertia)	m $x = 0$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = +m\frac{\partial^2 w}{\partial t^2}(0,t)$ $-EI\frac{\partial^2 w}{\partial x^2}(0,t) = 0$	m $x = l$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = -m\frac{\partial^2 w}{\partial t^2}(l,t)$ $EI\frac{\partial^2 w}{\partial x^2}(l,t) = 0$
End mass with moment of inertia (mass = m, moment of inertia = J_0)	m, J_0 $x = 0$ $-EI\frac{\partial^2 w}{\partial x^2}(0,t) = J_0\frac{\partial^3 w}{\partial x\partial t^2}(0,t)$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = +m\frac{\partial^2 w}{\partial t^2}(0,t)$	m, J_0 $x = l$ $EI\frac{\partial^2 w}{\partial x^2}(l,t) = +J_0\frac{\partial^3 w}{\partial x\partial t^2}(l,t)$ $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = -m\frac{\partial^2 w}{\partial t^2}(l,t)$

$+J_0\frac{\partial^3 w}{\partial x\partial t^2}(l,t)$

Figure 8.16 Boundary conditions for the transverse vibration of a beam.