

Design of damping layout using spatial-damping identification methods

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Abstract

The design of a damping layout can result in a frequency-focused reduction of vibration responses. Theoretical approaches that relate the spatial-damping parameters with the frequency content of the damping are limited. This research introduces a theoretical approach to damping-layout design (location and size) with frequency-content control. Initially, the frequency-response functions (measured or simulated) are modified to obtain the required damping layout via spatial-damping identification methods. The use of these methods provides a straightforward relationship between the frequency responses and the targeted spatial damping. The Lee-Kim spatial-damping identification method is used in the presented numerical and experimental case studies. The numerical and experimental results show that the approach is capable of providing the desired frequency content. This approach can be a valuable tool for a damping-layout assessment as high damping can be achieved with a reduced amount of damping material in a single-step solution.

1 Introduction

Damping is the dissipation of mechanical energy, mostly in the form of heat and, to a lesser extent, as acoustic radiation, transmission to coupled dynamic systems or other forms of dissipation [1]. In structural dynamics, damping, combined with mass and stiffness, represents the dynamic properties of a structure and is important for the validation and building of analytical/numerical models in civil, mechanical and aerospace engineering [2, 3].

In these industries, a number of structures are treated with damping materials to reduce the amount of structure-borne noise [4], to decrease vibration levels [1] or to increase fatigue life [5]. The industrial use of a damping treatment demands its optimization for reasons such as the cost-effectiveness and the mass loading of the structure. The result of this optimization approach should be the configuration of the damping layout with the minimum use of damping material – in short, its minimum spatial layout.

The standard approach to identifying damping in linear mechanical systems is to use one of the following methods: logarithmic decay [6] in the time domain, a continuous wavelet transform [7], the Morlet wave method [8] or the synchrosqueezed wavelet [9] in the time-frequency domain, or half-power point [6] and circle fit [6] in the frequency domain. It is also possible to evaluate the internal damping using macroscopic constitutive models [10]. However, these damping-identification methods do not provide any spatial information (i.e., the damping distribution throughout the structure).

For spatial damping, direct-damping identification methods were developed that identify the spatial damping directly from the frequency response functions (FRFs) without a transformation to the modal coordinates. Lee and Kim presented the dynamic-stiffness method [11], which identifies the damping separately from the mass and stiffness based on the imaginary and the real properties of the FRF. Other spatial-damping identification methods, not considered in this research, are reviewed in [12, 13, 14, 15, 16, 17].

Spatial-damping optimization approaches can be divided into the experimental and analytical [4]. The experimental approaches normally use laser vibrometry to map the vibration responses at several locations. These responses are subsequently examined and then the damping is applied to selected regions [1]. It is important to excite the structure over a wide frequency range in order to identify all the noise and transfer paths [4], which can be a time-consuming operation. On the other hand, the analytical approach consists of maximizing the damping or minimizing the structural responses by changing the numerical/analytical model parameters within the given constraints. The advantage of the analytical approach over the experimental approach is that it can be applied during the early stages of the design, but it is usually calculation-intensive and requires a detailed structural model (e.g., a large FEM model). There are a number of less general, spatial-damping optimization methods that are geometry- or material-specific (e.g., for plates [18, 23], shells [19], composite materials [20]). General material can be implemented into the FEM-based method [21, 22], but the result is a damping layout of variable thickness fragmented over the structure that is not very practical to implement.

In contrast to the optimization methods where typically the mass volume of the damping material is minimized, this research focuses on damping design for frequency-focused vibration reduction. The underlying idea is to use one of the existing spatial-damping identification methods that gives a straightforward relationship between the frequency responses and the targeted spatial damping.

This research is organized as follows. The damping-layout design approach is introduced in Section 2. In Section 3, the theoretical background of the Lee-Kim method is briefly presented. In Section 4, the validation of the approach is illustrated with two numerical examples and later the performance of the approach is tested with a real beam experiment. Finally, the conclusions are drawn in Section 5.

2 Design of damping layout

A frequency-domain design approach is presented here in which the frequency-response functions (FRFs) are modified and the resulting changes in amplitudes are estimated using established spatial-damping identification methods. Fig. 1 shows the required steps. The input data is the measured (or synthesized) FRF matrix $\mathbf{H}(\omega)$, after which the modal damping ratios are changed in the frequency domain to obtain the modified FRF matrix $\mathbf{H}_{\text{MOD}}(\omega)$. The spatial-damping identification method is applied to both FRF matrices to obtain the initial $\mathbf{D}_{\text{INIT}}$ and modified $\mathbf{D}_{\text{MOD}}$ spatial-damping matrices. The difference between the spatial-damping matrices is the damping layout.

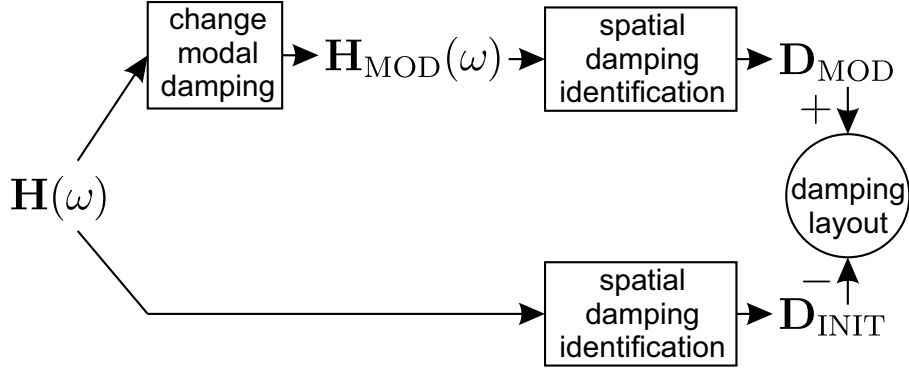


Figure 1: Proposed damping-layout design approach.

The input data $\mathbf{H}(\omega)$ can be synthesized from the spatial model [6]:

$$\mathbf{H}(\omega) = [\mathbf{K} - \omega^2 \mathbf{M} + i\mathbf{D}]^{-1} \quad (1)$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the hysteretic damping matrix and ω is the angular frequency. The second option is to synthesize $\mathbf{H}(\omega)$ from the modal data. The FRF matrix is synthesized for each coordinate j and k as the sum over n modes as [6]:

$$\mathbf{H}_{jk}(\omega) = \sum_{r=1}^n \frac{rA_{j,k}}{(1 + i\eta_r)\omega_r^2 - \omega^2} \quad (2)$$

where r is the mode number, $rA_{j,r}$ is the modal constant of the r -th mode for the matrix coordinates j and k , ω_r is the eigenfrequency of the r -th mode and η_r is the damping ratio of the r -th mode.

After obtaining the initial FRFs, the damping ratios of the selected modes are changed to obtain the desired frequency content, see Fig 2. Regardless of the input data (e.g., measured or synthesized) the modal parameters of the initial FRF matrix can be extracted using experimental modal analysis (EMA) [6]. The mode-based approach to obtaining the desired frequency content is preferred because the vibration responses are sensitive to the damping changes for the frequency range around the resonances only [1]. From the modified modal parameters (i.e., the damping ratio changes) the modified FRF matrix is reconstructed with (2).

Finally, the spatial-damping identification method is used to identify the spatial-damping matrices from both FRF matrices. The identified spatial-damping matrix is the spatial distribution of the damping over the structure and the difference between the initial and modified damping matrices is the required damping layout.

The proposed spatial-damping design approach can be developed into an iterative one to account for the mass and stiffness changes of the applied damping treatment [23], but its development is beyond the scope of this research.

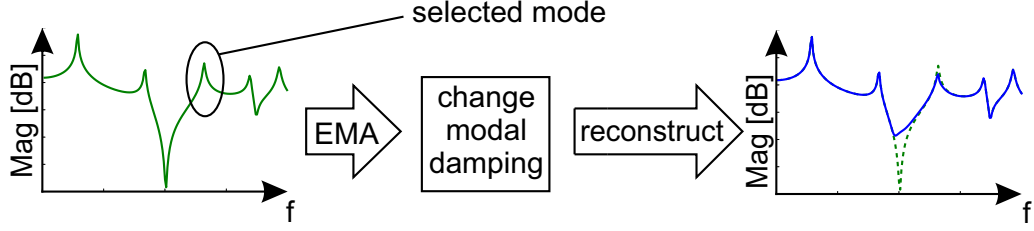


Figure 2: Preparation of the FRFs for the identification step.

The Lee-Kim [11] spatial-damping identification method will be used in the case studies. The method is general and can be applied to any type of structure; its performance was thoroughly analysed in [24]. A theoretical presentation of the method is given next.

3 Spatial-damping identification method

In this section the background of the Lee-Kim [11] direct-damping identification method for hysteretic damping is briefly presented. Assuming a linear system and a harmonic excitation/response, the general, second-order, matrix differential equation can be written in the frequency domain as [6]:

$$[\mathbf{K} - \omega^2 \mathbf{M} + i \mathbf{D}] \mathbf{X}(\omega) = \mathbf{F}(\omega) \quad (3)$$

From (3), the receptance FRF matrix $\mathbf{H}(\omega)$ is defined as [6]:

$$\mathbf{X}(\omega) = [\mathbf{K} - \omega^2 \mathbf{M} + i \mathbf{D}]^{-1} \mathbf{F}(\omega) = \mathbf{H}(\omega) \mathbf{F}(\omega) \quad (4)$$

and the dynamic stiffness matrix $\mathbf{Z}(\omega)$ is defined as the matrix inverse of $\mathbf{H}(\omega)$ at each frequency point ω :

$$\mathbf{Z}(\omega) = \mathbf{H}(\omega)^{-1} = [\mathbf{K} - \omega^2 \mathbf{M} + i \mathbf{D}] \quad (5)$$

Using (5) the hysteretic damping matrix might be obtained directly from the imaginary part of the dynamic stiffness matrix $\mathbf{Z}(\omega)$:

$$\text{imag}(\mathbf{Z}(\omega)) = \text{imag}([\mathbf{H}(\omega)]^{-1}) = \mathbf{D}, \quad (6)$$

Rearranging (6) to isolate the damping matrix $\mathbf{D}$ gives:

$$\mathbf{D} = \text{imag}([\mathbf{H}(\omega)]^{-1}) \quad (7)$$

Method (7) is not limited to hysteretic damping [25].

4 Numerical 5 DoF case study

Fig 3 represents a 5-degree-of-freedom (DoF) lumped-mass model that will be used for the initial validation of the proposed method. Two model properties are defined by the mass $m = 5$ kg and the stiffness $k = 2 \cdot 10^6$ N/m, and are arranged into mass $\mathbf{M}$ and stiffness $\mathbf{K}$ matrices. The initial hysteretic spatial-damping values d of the model are defined as the stiffness-proportional damping [6] at the matrix level as:

$$\mathbf{D} = \beta \mathbf{K}, \quad (8)$$

where β is the stiffness proportional constant, which was chosen to be 0.01.

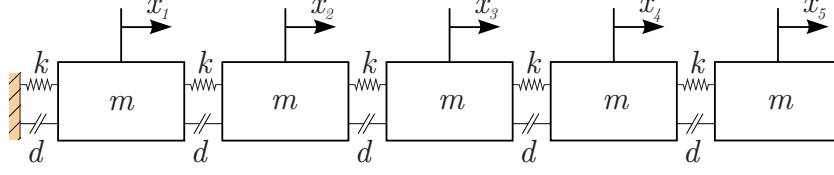


Figure 3: 5-DoF model used in the simulation case.

4.1 FRF matrix modification

With the defined structural matrices $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{D}$ the full FRF matrix can be obtained using (1). To modify the FRF matrix the modal parameters are extracted from the spatial model. The following eigenproblem has to be solved [6]:

$$[\mathbf{K} + i\mathbf{D} - \lambda_r^2 \mathbf{M}] \boldsymbol{\psi}_r = \mathbf{0} \quad (9)$$

where λ_r is the r -th complex eigenvalue and $\boldsymbol{\psi}_r$ is the corresponding mode shape. The complex eigenvalue contains the information about the r -th eigenfrequency ω_r and the r -th damping ratio η_r [6]:

$$\lambda_r^2 = \omega_r^2 (1 + i\eta_r) \quad (10)$$

Stiffness proportional hysteretic damping is a special case where the modal damping η_r is equal to the proportional constant β [6]:

$$\eta_r = \beta \quad (11)$$

The FRF matrix $\mathbf{H}$ can now be written with the modal parameters as:

$$\mathbf{H}_{jk}(\omega) = \sum_{r=1}^n \frac{rA_{j,k}}{(1 + i\eta_r)\omega_r^2 - \omega^2} \quad (12)$$

where $rA_{j,k}$ is the modal constant that contains the product of the j -th and k -th component of the mode-shape vector:

$$rA_{j,k} = \psi_{r,j} \psi_{r,k} \quad (13)$$

In the 5-DoF case the damping ratio of the first and second modes was changed to 0.04 and then the FRFs were obtained with Equation (12), see the approach defined in Section 2. Fig 4 shows an example of the initial and the modified receptance magnitude FRF of $H_{2,3}(f)$, where 2-3 denotes that the structure was excited for the 2nd DoF and the responses were obtained for the 3rd DoF (this designation will be used throughout the paper).

4.2 Design of damping layout

Fig. 5 shows the values of the identified hysteretic damping matrix of the 5-DoF model using the hysteretic Lee-Kim method (7) where: (a) is the identified hysteretic damping matrix $\mathbf{D}$ from the $\mathbf{D}_{\text{INIT}}$, (b) is the identified hysteretic damping matrix $\mathbf{D}_{\text{MOD}}$ from the $\mathbf{D}_{\text{MOD}}$ and (c) is the difference between the two damping matrices. Larger absolute numerical values (e.g., $D_{1,1} = 45000$ N/m in Fig. 5), represent areas of higher damping. To obtain the damping layout the criteria for the most effective damping locations were selected. Fig. 6 shows the locations of the difference matrix Fig. 5(c) where the absolute damping values are higher than the selected threshold, in our case 65%:

$$\mathbf{D}_{\text{DL}} = [\text{difference} > \text{threshold}] \quad (14)$$

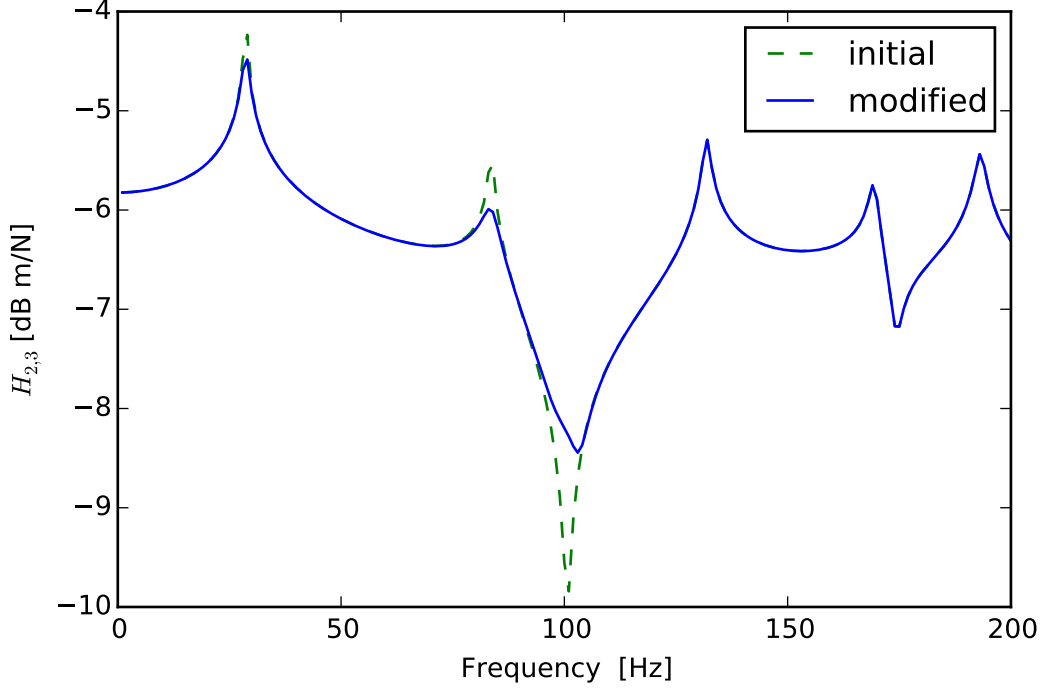


Figure 4: Initial and modified FRF for the 5-DoF model.

where $\mathbf{D}_{\text{DL}}$ is the damping-layout matrix.

To double-check the proposed damping layout the FRFs were reconstructed using the identified hysteretic damping matrix $\mathbf{D}_{\text{IDE}}$:

$$\mathbf{H}_{\text{REC}}(\omega) = \frac{1}{[\mathbf{K} - \omega^2 \mathbf{M} + i \mathbf{D}_{\text{IDE}}]} \quad (15)$$

The resulting FRF is shown in Fig. 7. The modified and the reconstructed FRF fit to each other.

5 Beam case study

Initially, numerically simulated data for the free-free beam is used to demonstrate the effectiveness of the Lee-Kim method. The damping layout is given as the result in the simulation step. In the validation step, the proposed damping layout was applied in a real experiment to a beam with the same properties as in the numerical simulations. The FRFs were measured and compared to the numerically modified ones.

5.1 Numerical simulations

The beam properties used for the numerical simulations were: density $\rho = 7850 \text{ kg/m}^3$, constant cross-section $h \times b = 1 \text{ mm} \times 30 \text{ mm}$, length $l = 400 \text{ mm}$ and Young's modulus $E = 210,000 \text{ MPa}$. The beam dimensions were selected to have a low modal overlap and to have a large number of modes in the frequency span up to 2000 Hz. The modal vectors and values were simulated using the Euler-Bernoulli theory [26]. The initial damping of the model is defined as the constant modal damping ratio of $\eta = 0.002$ for each mode as the hysteretic damping ratios for the bending vibrations of steels η_{steel} range from 0.002 to 0.006 [27]. The damping ratios for the modes 4 to 10 were increased from 0.002 to 0.02, and then the FRFs were resynthesized using (2), see Fig. 8.

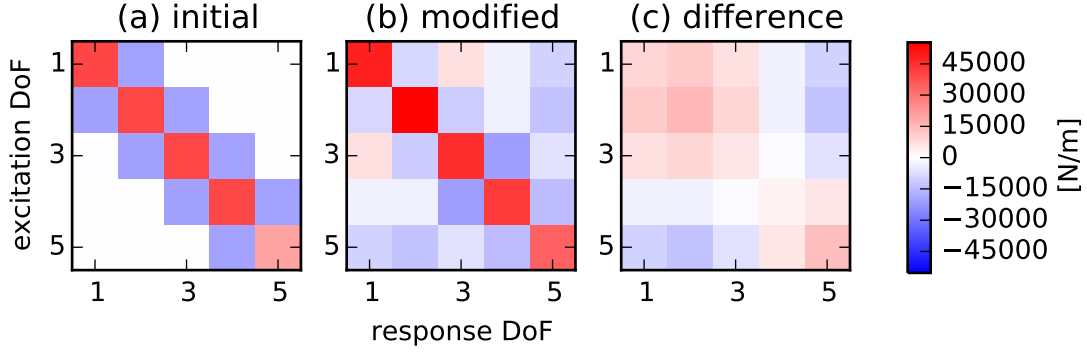


Figure 5: Hysteretic damping matrix $\mathbf{D}$: (a) simulated stiffness-proportional, (b) identified after FRF modification and (c) difference (b)-(a).

5.2 Design of damping layout

Fig. 9 shows the values of the identified hysteretic damping matrices of the beam model using the hysteretic Lee-Kim method (7) where: (a) is the identified hysteretic damping matrix from the initial FRFs, (b) is the identified hysteretic damping matrix from the modified FRFs and (c) is the difference between the two damping matrices.

To obtain the damping layout the criteria for the most effective damping locations were selected. Fig. 10 shows the locations of the difference matrices, while Fig. 9(c) shows where the absolute damping values are higher than the selected threshold (65% was used). DoFs 5 to 8 were selected as the proposed locations for the damping layout to cover most of the high damping areas (excitation-response DoF pairs: 5-5, 5-8, 8-5 and 8-8) found in Fig. 9(c). The damping terms close to the main diagonal connect the neighbouring DoF and form a continuous area to apply the damping treatment.

6 Experimental validation of the beam case

The proposed damping layout from Section 5.2 is here applied to the real beam to analyse the performance of the proposed method. Damping over the proposed locations was achieved using the established constrained-layer damper configuration [1]. The selection of the damping-treatment design parameters (e.g., the material type or thickness) to obtain the desired damping values is beyond the scope of this research. The interested reader is referred to [1, 28, 29].

6.1 Sample preparation

The experiment was conducted on the two equal-sized, free-free, beam specimens to validate the damping-layout result. The first specimen was a plain sample (without any damping treatment), while the second was treated with constrained-layer damping (CLD) at the proposed locations, see Fig. 11. Two soft springs (stiffness ≈ 50 N/m) were used at each beam boundary in the y-direction to limit the rigid-body translation after the impact. Isolated DoFs (e.g., 2-2 and 11-11) were not considered as a viable option – applying the constraining layer locally over a small area is not effective [1]. The visco-elastic layer for the application was 3M 112P02 damping material [30]. The steel constraining layer was of the same material and thickness as the beam to maximize the damping [30]. Holes were drilled in the constraining layer to measure the responses of the beam only.

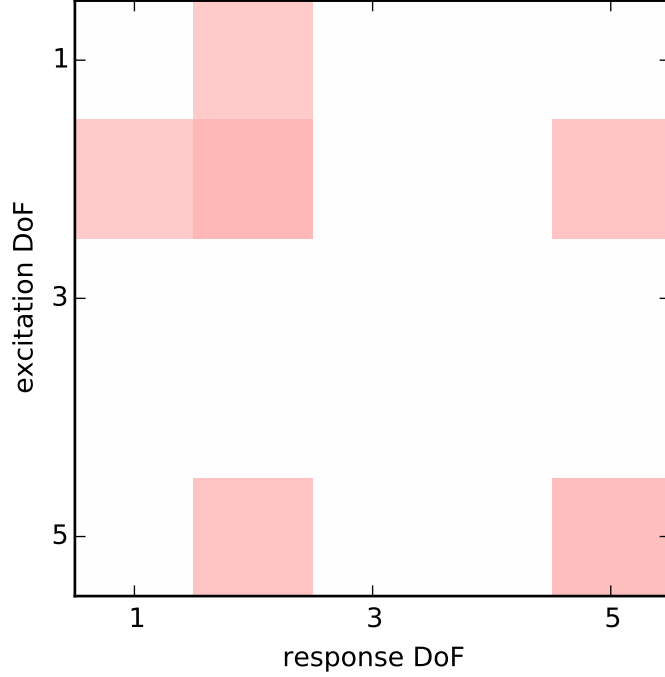


Figure 6: Identified damping layout at 65% threshold.

6.2 Measurement setup

The measurement setup is shown in Fig. 12. A custom-made solenoid impactor with a PCB 086E80 force sensor was used for the repeatable impulse excitation. The response (velocity) measurements employed a Polytec PDV100 laser vibrometer. This impactor/laser-based measurement allows for a non-contact measurement without structural modification due to the added stiffness or mass from the sensors or shakers. The data acquisition and signal processing made use of a custom python-software environment using the pyDAQmx library [31] to interface the NI 9215 acquisition hardware. The sampling rate was 100 kHz and the signal was captured for 5 seconds.

To obtain a full FRF matrix $\mathbf{H}$ the beam was sequentially excited at 15 points, and the responses were measured at the same 15 points (15×15 excitation-response pairs, $n = 15$), as shown in Fig 12. Each excitation-response point was measured three times to obtain the averaged H_1 estimator (mobility FRF) and later divided by $i\omega$ in the frequency domain to obtain the receptance FRFs [6].

6.3 Measured frequency responses

Fig. 13 shows the comparison of the measured receptance magnitude FRF for the CLD-damped beam with the modified one. It is clear that the damping affects the frequency range from the 5-th mode up, whereas in the FRF preparation step the damping was increased from the 4th mode up. In addition, the added stiffness and mass due to the damping treatment were not considered during the damping-layout design step, thus slight amplitude and mode eigenfrequencies changes are observable in Fig. 13.

7 Conclusion

This research introduces a theoretical approach to damping-layout design with frequency-content control. The approach is based on the spatial-damping identification methods and can be applied

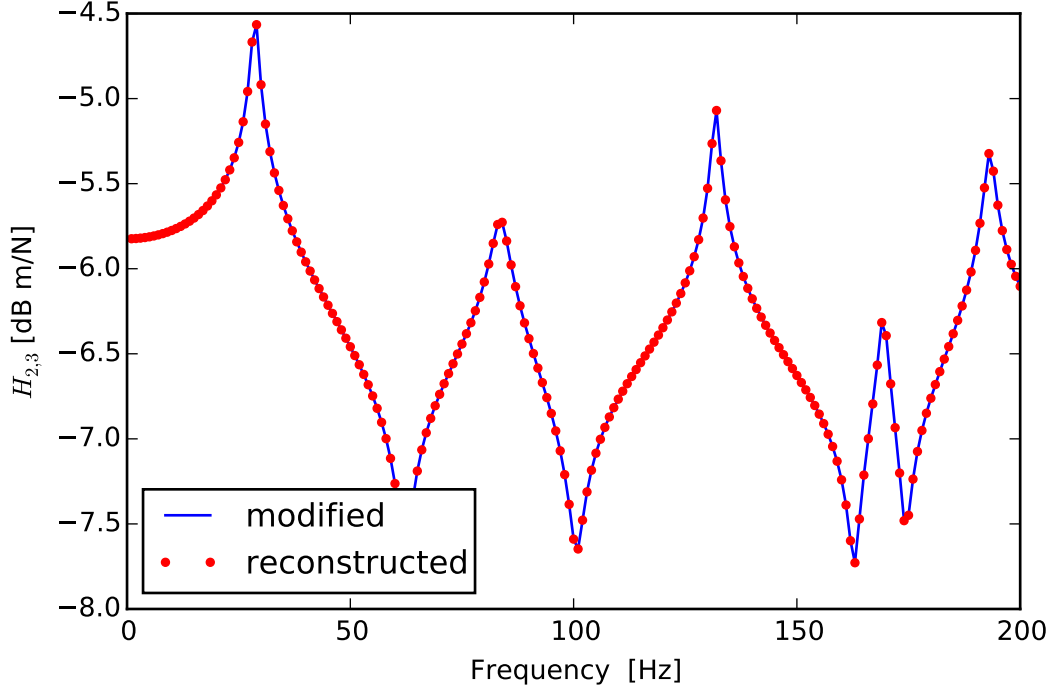


Figure 7: Reconstructed FRF for 5-DoF model with artificially damped modes 1 and 2 at $H_{2,3}(f)$.

to general structures. First, the damping-layout design approach is summarised and, second, for validation purposes, the approach is analysed for two cases: a 5-degree-of-freedom (DoF) model and a larger 15-DoF beam numerical model. Lastly, the performance of the approach is demonstrated on real beam test cases, from which the following conclusions can be drawn:

- The approach is capable of providing the desired frequency content.
- High damping is achieved with a reduced amount of damping material.
- The approach is simple and flexible: the input data can be measured, modelled or even a mixture of both.
- The use of a damping model and a damping-identification method is open to the user, while for specific cases a more advanced damping model can be used.

8 Acknowledgements

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Bibliography

References

- [1] D.J. Mead. *Passive Vibration Control*. Wiley, 1998.
- [2] M.I. Friswell and J.E. Mottershead. *Finite Element Model Updating in Structural Dynamics*. Kluwer, 1995.

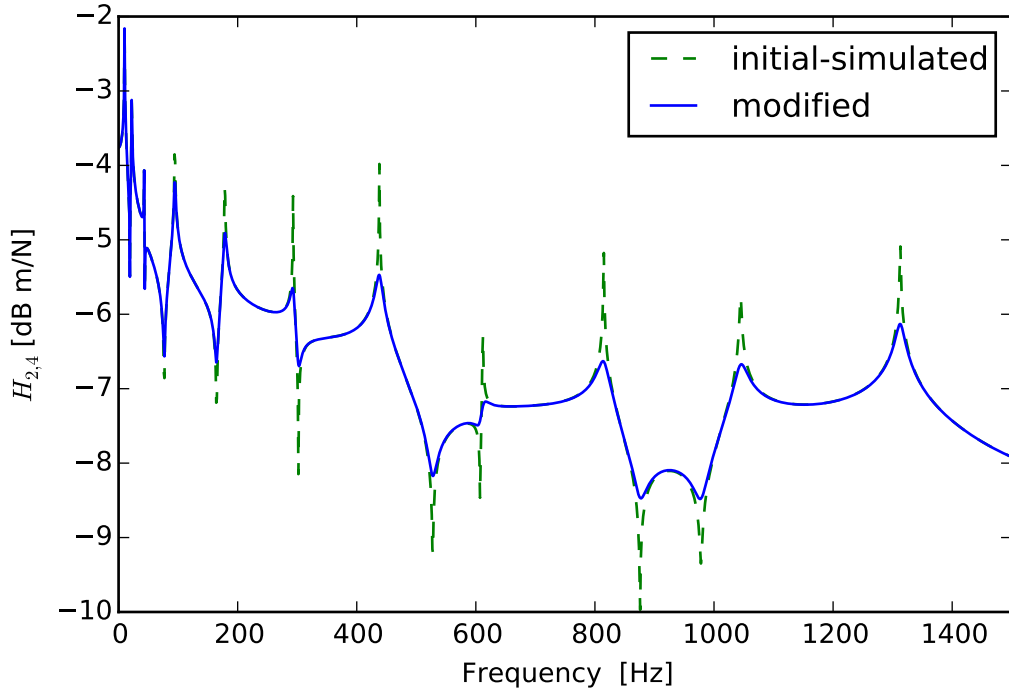


Figure 8: Example of beam FRF used in simulation case.

- [3] J.E. Mottershead, M. Link, and M.I. Friswell. The sensitivity method in finite element model updating: A tutorial. *Mechanical Systems and Signal Processing*, 25(7):2275–2296, 2011.
- [4] S. Subramanian, R. Surampudi, K. R. Thomson, and S. Vallurupalli. Optimization of Damping Treatment for Structure Borne Noise Reduction. SAE Technical Paper 2003-01-1592, SAE Technical Paper, 2003.
- [5] M. Česnik, J. Slavič, and M. Boltežar. Uninterrupted and accelerated vibrational fatigue testing with simultaneous monitoring of the natural frequency and damping. *Journal of Sound and Vibration*, 331(24):5370–5382, 2012.
- [6] D.J. Ewins. *Modal testing: theory, practice and application*, 2nd ed. Research Studies Press, 2000.
- [7] J. Slavič, I. Simonovski, and M. Boltežar. Damping identification using a continuous wavelet transform: application to real data. *Journal of Sound and Vibration*, 262(2):291–307, 2003.
- [8] J. Slavič and M. Boltežar. Damping identification with the Morlet-wave. *Mechanical Systems and Signal Processing*, 25(5):1632–1645, 2011.
- [9] M. Mihalec, J. Slavič and M. Boltežar. Synchrosqueezed wavelet transform for damping identification. *Mechanical Systems and Signal Processing*, 80:324–334, 2016.
- [10] P. Jana and A. Chatterjee. An internal damping formula derived from dispersed elastoplastic flaws with Weibull-distributed strengths. *International Journal of Mechanical Sciences*, 87:137–149, 2014.
- [11] J.-H. Lee and J. Kim. Development and Validation of a New Experimental Method to Identify Damping Matrices of a Dynamic System. *Journal of Sound Vibration*, 246:505–524, 2001.

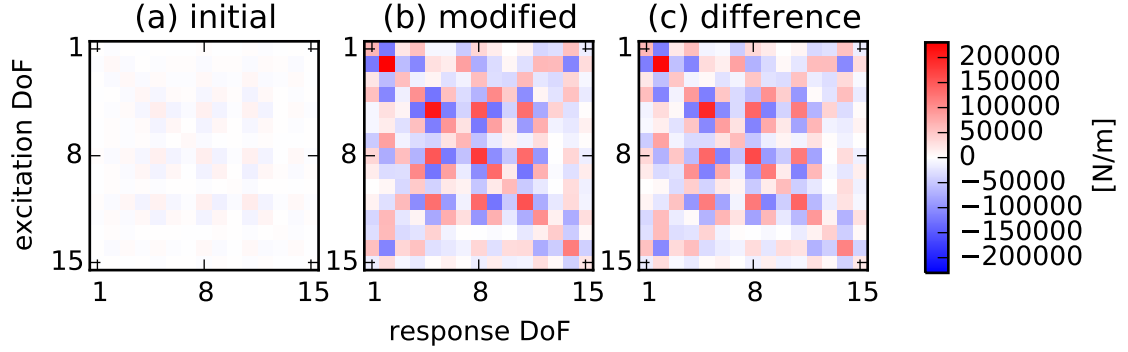


Figure 9: Hysteretic damping matrices for beam case.

- [12] D.F. Pilkey and D.J. Inman. A survey of damping matrix identification. In *16th International Modal Analysis Conference*. SEM, 1998.
- [13] A. Srikantha Phani and J. Woodhouse. Viscous damping identification in linear vibration. *Journal of Sound and Vibration*, 303(3–5):475 – 500, 2007.
- [14] A. Srikantha Phani and J. Woodhouse. Experimental identification of viscous damping in linear vibration. *Journal of Sound and Vibration*, 319(3–5):832 – 849, 2009.
- [15] H. Öztürk and J.G. McDaniel. Spatial mapping of modal damping in vibrating plates. *The Journal of the Acoustical Society of America*, 126(4):2244–2244, 2009.
- [16] M. Prandina, J.E. Mottershead, and E. Bonisoli. An assessment of damping identification methods. *Journal of Sound and Vibration*, 323(3–5):662 – 676, 2009.
- [17] Q. Leclère, F. Ablitzer, and C. Pézerat. Practical implementation of the corrected force analysis technique to identify the structural parameter and load distributions. *Journal of Sound and Vibration*, 351:106–118, 2015.
- [18] T. Lassila. Optimal damping of a membrane and topological shape optimization. *Structural and Multidisciplinary Optimization*, 38(1):43–52, 2008.
- [19] S.Y. Kim, C.K. Mechefske, and I-Y. Kim. Optimal damping layout in a shell structure using topology optimization. *Journal of Sound and Vibration*, 332(12):2873–2883, 2013.
- [20] M. A. Trindade. Optimization of active–passive damping treatments using piezoelectric and viscoelastic materials. *Smart Materials and Structures*, 16(6), 2007.
- [21] A. Lumsdaine and R. A. Scott. Shape optimization of unconstrained viscoelastic layers using continuum finite elements. *Journal of Sound and Vibration*, 216(1):29–52, 1998.
- [22] H. Koruk and K.Y. Sanliturk. Optimisation of damping treatments based on big bang–big crunch and modal strain energy methods. *Journal of Sound and Vibration*, 333(5):1319–1330, 2014.
- [23] T. Yamamoto, T. Yamada, K. Izui, and S. Nishiwaki. Topology optimization of free-layer damping material on a thin panel for maximizing modal loss factors expressed by only real eigenvalues. *Journal of Sound and Vibration*, 358:84–96, 2015.
- [24] M. Brumat, J. Slavič, and M. Boltežar. Spatial damping identification in the frequency domain—A theoretical and experimental comparison. *Journal of Sound and Vibration*, 376:182–193, 2016.

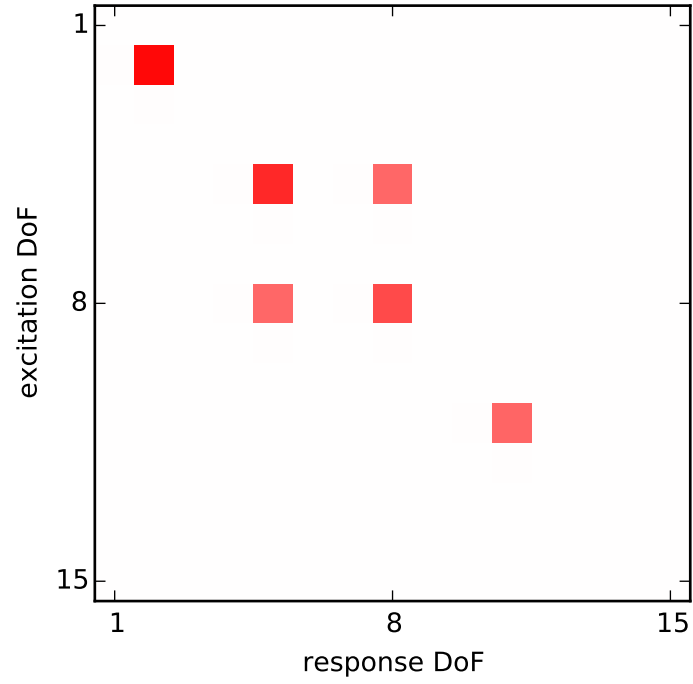


Figure 10: Identified damping layout at 65% threshold.

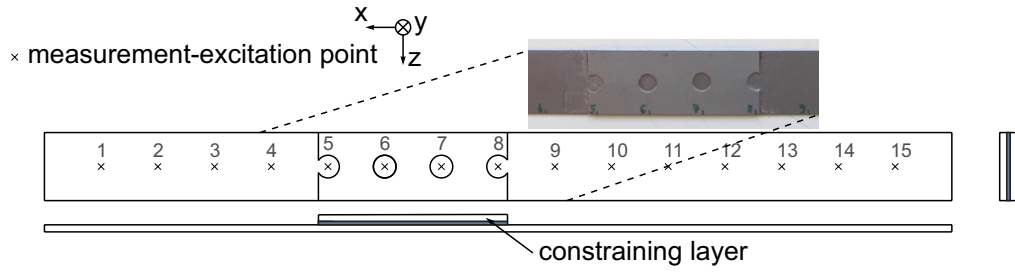


Figure 11: Beam with added constrained-layer damper.

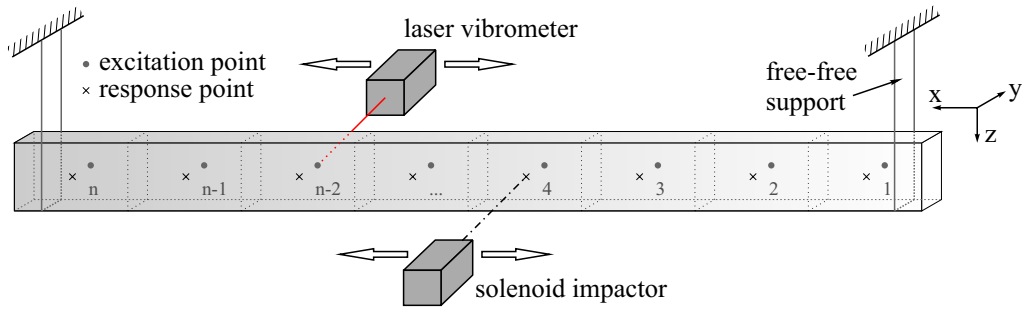


Figure 12: Measurement setup.

- [25] G.O. Ozgen and J.H. Kim. Direct identification and expansion of damping matrix for experimental-analytical hybrid modeling. *Journal of Sound and Vibration*, 308(1–2):348–372, 2007.

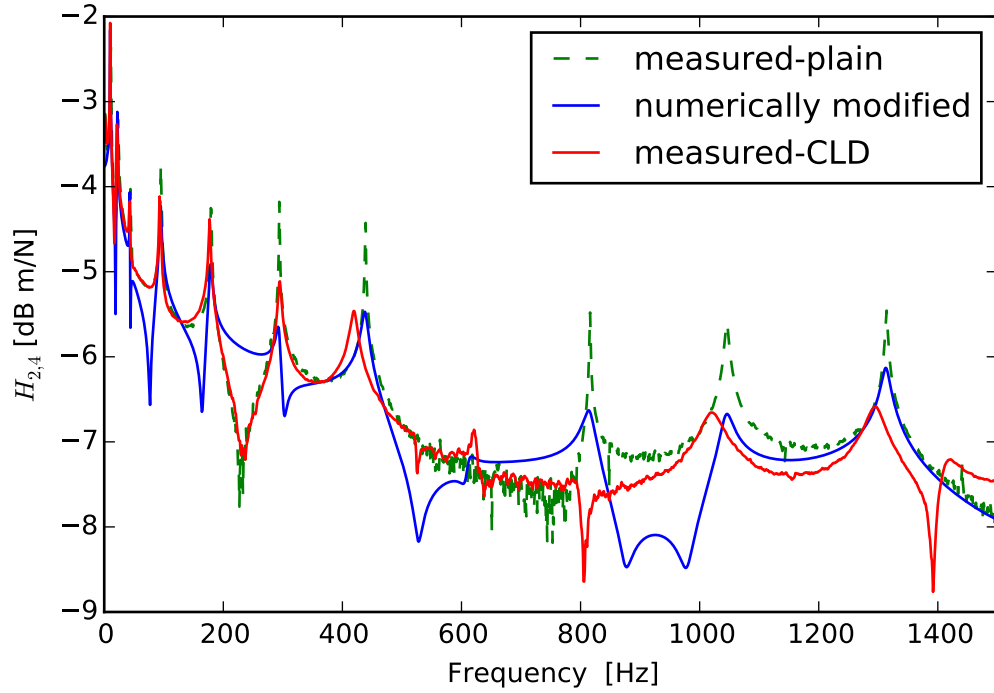


Figure 13: The comparison of numerically modified and measured FRF.

- [26] Rao, S.S. *Mechanical Vibrations*, 4th ed. Pearson Education, 2003.
- [27] B.J. Lazan. *Damping of materials and members in structural mechanics*. Pergamon Press, 1968.
- [28] A.D. Nashif, D.I.G. Jones, and J.P. Henderson. *Vibration Damping*. Wiley, 1985.
- [29] M. W. L. M. Rijnen, F. Pasteuning, R. H. B. Fey, G. van Schothorst, and H. Nijmeijer. A numerical and experimental study on viscoelastic damping of a 3D structure. *Journal of Sound and Vibration*, 349:80–98, 2015.
- [30] 3MTM viscoelastic damping polymeres 112-130 technical data, 2012.
- [31] P. Cladé. *pyDAQmx: a Python interface to the National Instruments DAQmx driver*, 2015.