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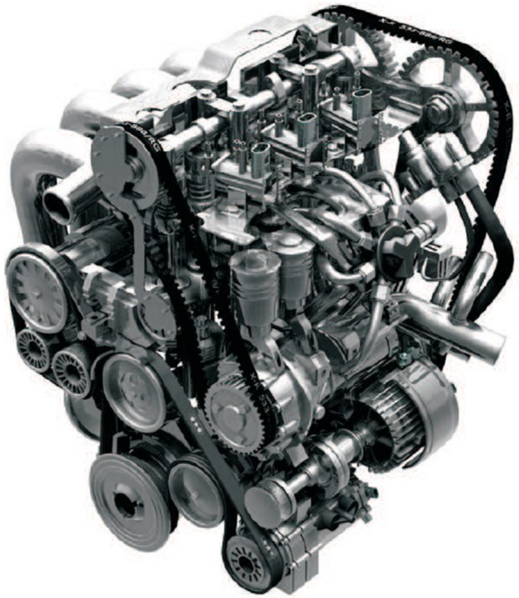
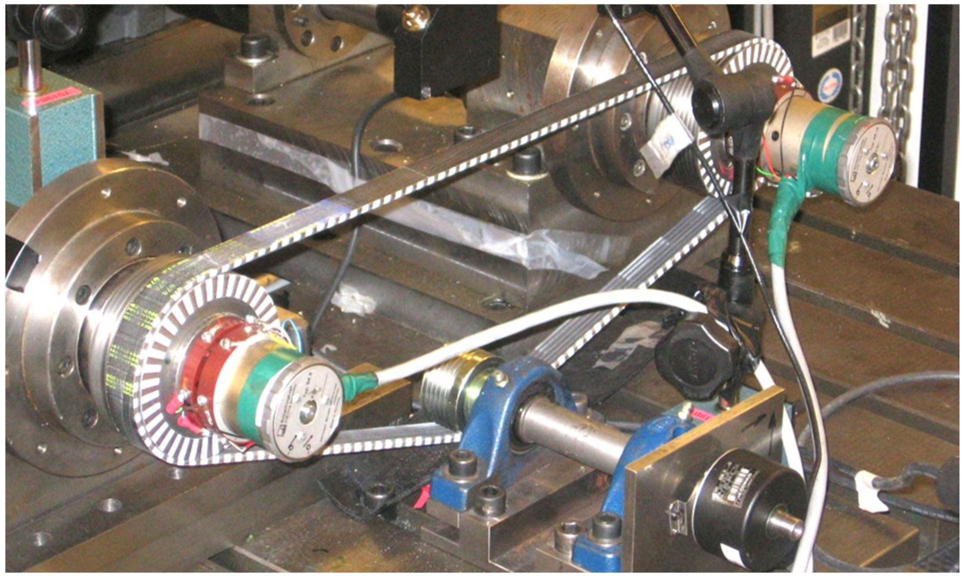
A1: Purpose, Ultimate objective

- ***Understanding*** and ***modelling*** the ***belt drive***'s dynamics in order to **minimize vibrations** and **noise**.





Objectives - scope of work

- ***Belt drive:***
  - Search for an appropriate belt model and the model verification
  - Analysing and modelling belt-pulley contact problems
  - Modelling and verifying the model of the whole drive
  - Drive's model fine tuning
  - Model analysis

A5: Quantification of Project benefits

- Dynamical systems' models enables us to master vibrations and consecutively noise.
- Virtual dynamical analysis of the belt drive at the drive's offering stage.
- Shorter product's development period.
- Student involvements, applied scientific research

STATE OF THE ART

and

PREVIOUS COOPERATION

B0: State of the Art – Belts (1)

There are different objects on which the dynamical analysis is orientated to:

- the belt itself [2,4,14,15,31,33,35,36,38,41,42,45,47,49],
- the pulley [8,12,51],
- the contact between belt and pulley [21,24,25,32,44,48,50],
- whole belt drive [1,3,5,6,7,9,10,11,12,13,16,18,19,20,22,23,26,27,28,29,30,34,37,41,40,43] and
- belt drive with multiple belts [17].

B0: State of the Art – Belts (2)

There are different models used to model the belt:

- the continuous viscoelastic dynamical model [14,15],
- the continuous elastic dynamical model [3,4,5,7,9,10,23,27,31,33,35,36,37,38,41,40,47],
- the discrete dynamical model [1,11,17,21,41],
- the FEM model [12,26,40].

B0: State of the Art – Belts (3)

The continuous belt model for dynamical research is used from ~1960 onwards. It may be linear or non-linear elastic model or viscoelastic one.

The articles tackling the dynamical problem with the FEM are scarce.

There is only one article that solves timing belt model and contact between pulley and belt in purely discrete way [21].

B0: State of the Art – Belts (4)

Modeling the belt alone is not sufficient to capture whole belt-system dynamics. The effects of:

- the free span on system dynamics,
- the pulley eccentricity,
- the tensioner dynamics and
- dynamic stability

are studied when whole belt-drive is modeled.

B0: State of the Art – Belts (5)

Some conclusions:

- The smaller the radius of the rounded tip corners of the pulley teeth, the lower the noise of the belt [43].
- Small traverse oscillation of an endless band supported by wheels couples the response of the free spans of the band to oscillation of the wheels [37,40].
- Adjustment of pulley tooth height is confirmed to be effective for reducing the traverse vibration [18].

B0: State of the Art – Belts (6)

Some conclusions (cont.):

- High frequency noise is generated by the discontinuous slips and the flow of holding air between the belt and pulley of trial belts [13].
- The quadratic non-linerity terms in the equations of motion of belt drive are found to affect the belt drive systems significantly [10].

B0: State of the Art – Belts (7)

Some conclusions (cont.):

- The vibrational power of the two belt-spans flows into the tensioner [9].
- The belt vibrates laterally if the accessory belt's natural frequency is equal to ½ of the frequency of belt vibration length fluctuation [5].

B0: State of the Art – Belts (8)

Some conclusions (cont.):

- Transverse belt instability mechanisms:
  - tensioner resonance [41],
  - belt resonance [41],
  - belt critical speed [41],
  - Mathieu instability due to belt tension variations [3,7,41,49]

B0: Previous cooperation (1)

Mihael Bogataj, ***Vibration and noise of automotive belts***,
Diploma Thesis, 2000,
University of Ljubljana, Faculty of Mechanical Engineering

BELTS

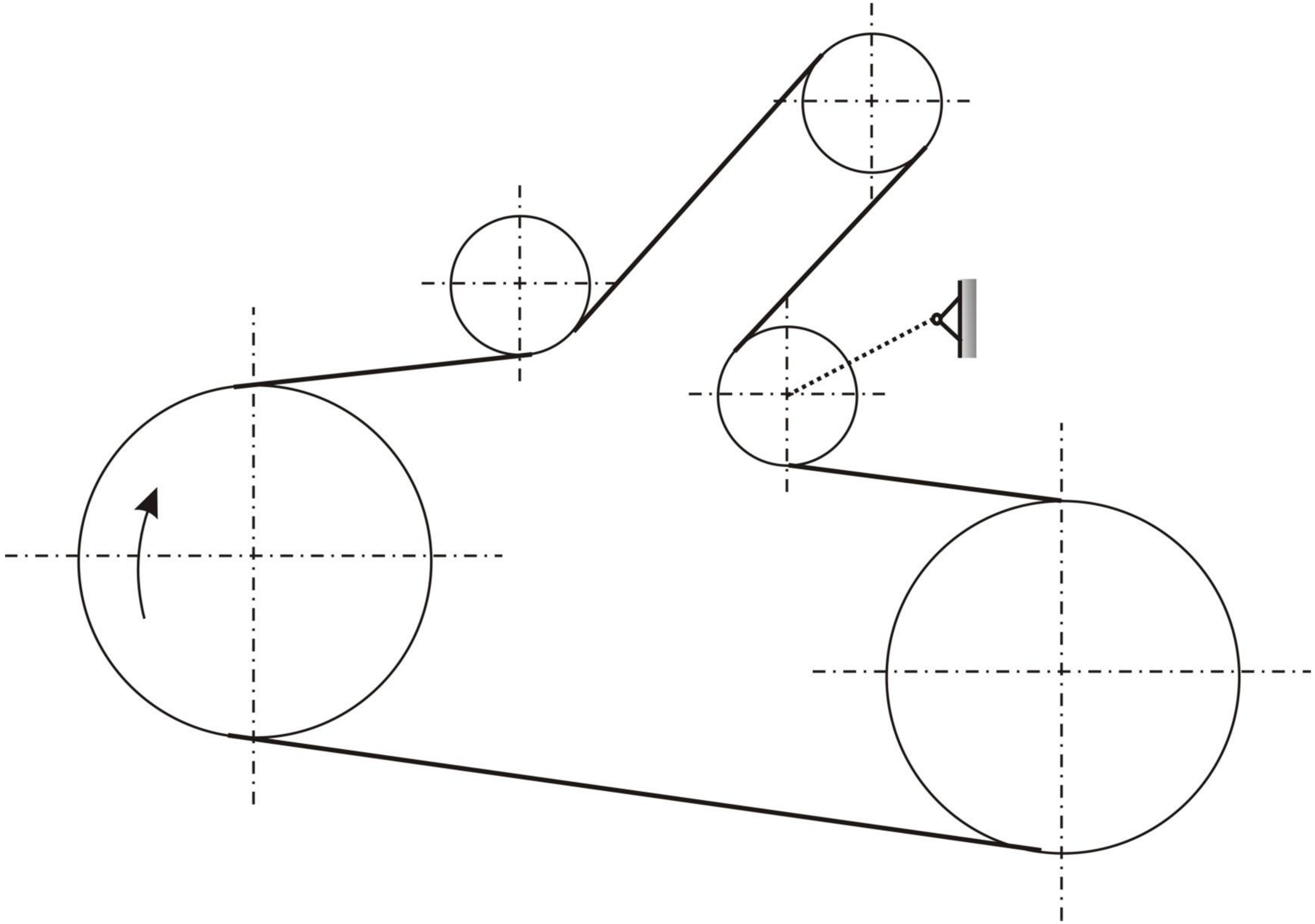
and

BELT DRIVES

B1: Analytical
Belt-Drive vibrations

- Torsional vibrations of pulleys.
- Vibration due to pulleys' elastic mounting.
- Belt span vibrations:
  - axial vibrations,
  - transversal vibrations (in-plane and out-of-plane),
  - torsional vibrations,
  - couplings between ones.

B1: Analytical - Belt-Drive vibrations

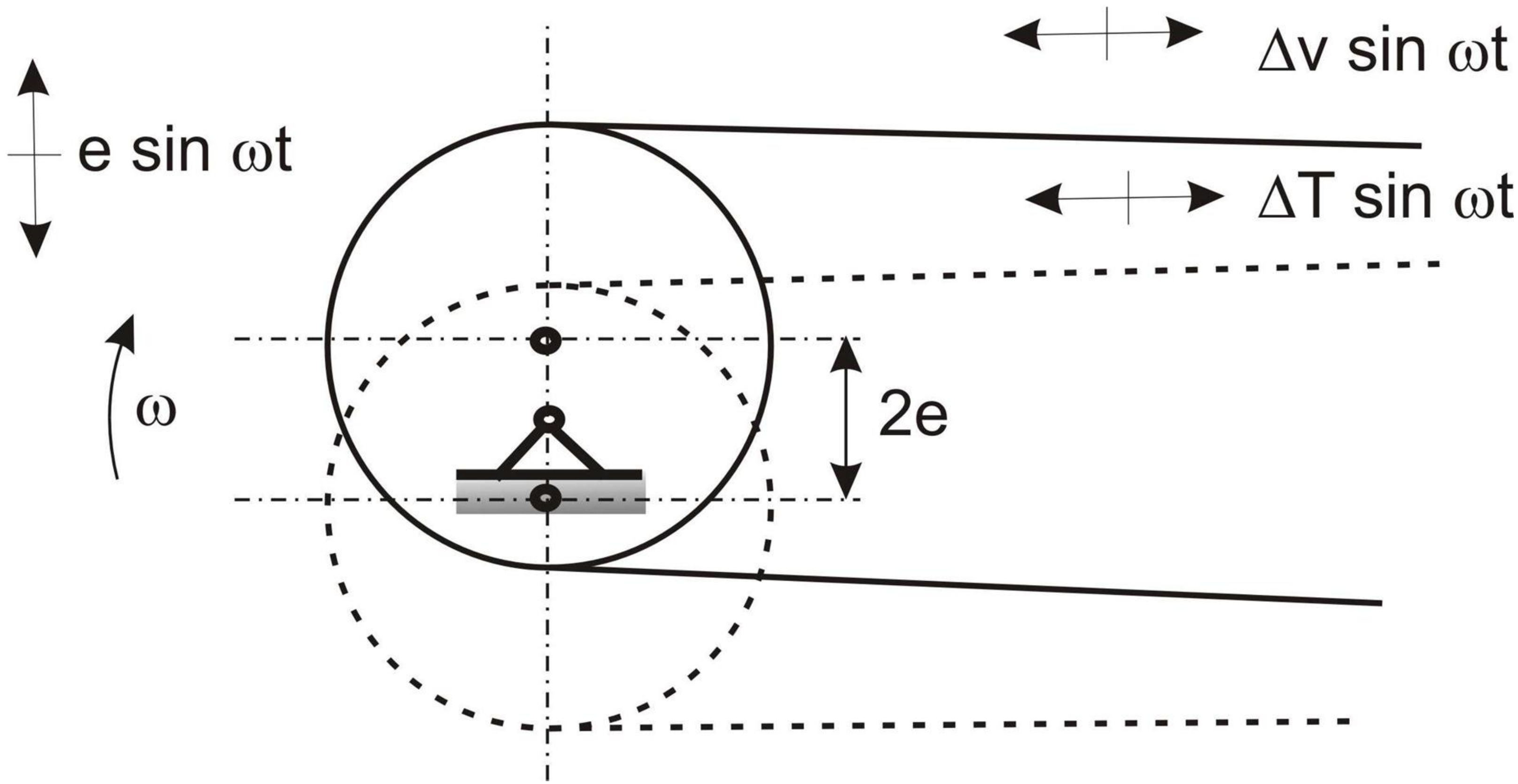


B1: Analytical - Excitation mechanisms for transverse vibration

- No direct forces applied on the belt span
- Non-homogeneous boundary conditions due to pulley's eccentricity
- Variation of the belt velocity or/and belt tension (parametric exitation) due to:
  - axial belt oscillations,
  - variation of the driving torque,
  - pulley's eccentricity,
  - tensioner dynamics,
  - temperature variations,
  - elasticity of pulleys' mountings.

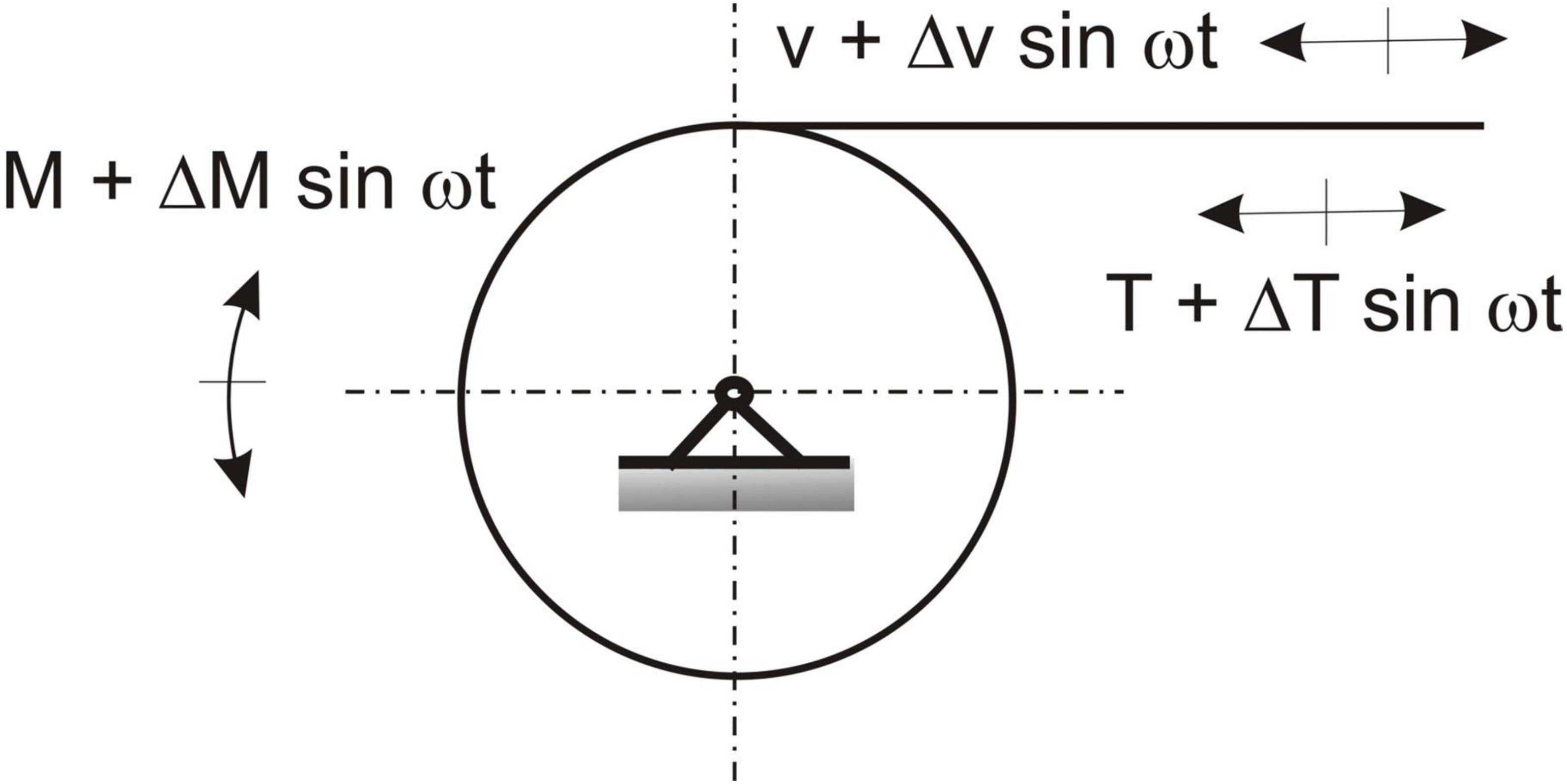
B1: Analytical - Excitation mechanisms for transverse vibration

- Pulley's eccentricity



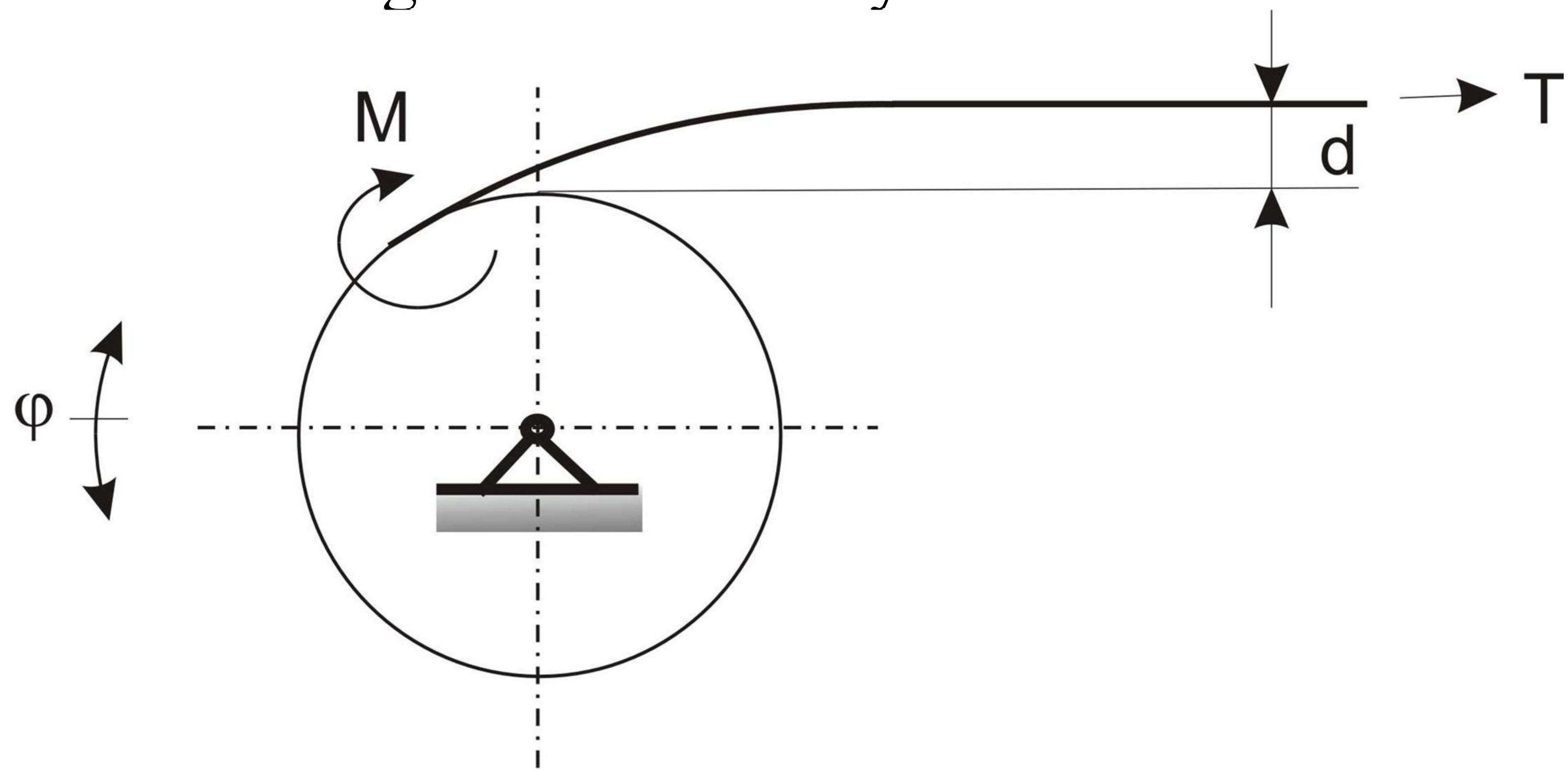
B1: Analytical - Excitation mechanisms for transverse vibration

- External moment



B1: Analytical - Excitation mechanisms for transverse vibration

- Non-homogeneous boundary conditions



B1: Analytical
Non-Linear effects

- are the consequence of
  - high belt velocity ($v > 0.3\ c$),
  - coupled transverse and axial vibrations,
  - parametric exitation and parametric resonances,
  - material models (viscoelastic),
  - tensioner dynamics.

B1: Analytical
Belt-Span Linear Equation of motion

- Free transverse vibrations of the string model:

$$\left(c^2 - v^2\right)\frac{\partial^2 w}{\partial x^2} - 2v\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial^2 w}{\partial t^2} - b\left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right) = 0$$

- Free transverse vibrations of the beam model:

$$-a^2\frac{\partial^4 w}{\partial x^4} + \left(c^2 - v^2\right)\frac{\partial^2 w}{\partial x^2} - 2v\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial^2 w}{\partial t^2} - b\left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right) = 0$$

B1: Analytical
Belt-Span Coupled Equations of motion

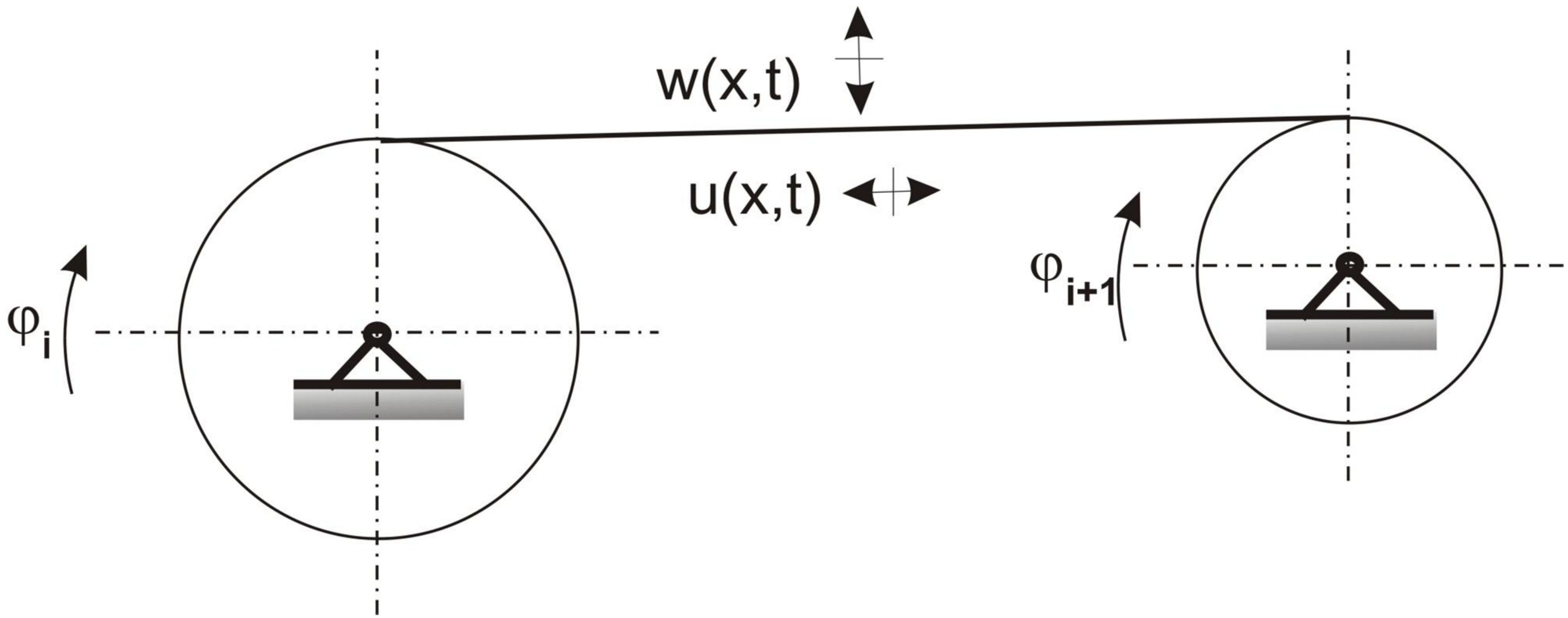
- Free axial-transverse vibrations of the model:

$$-a^2\frac{\partial^4 w}{\partial x^4}+\left(c^2-v^2\right)\frac{\partial^2 w}{\partial x^2}-2v\frac{\partial^2 w}{\partial x\,\partial t}-\frac{\partial^2 w}{\partial t^2}-b_w\left(\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial x}\right)-\left(c^2-f^2\right)\frac{\left(1+\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}-\left(1+\frac{\partial u}{\partial x}\right)\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial x^2}}{\left[\left(1+\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]^{3/2}}=0$$

$$\left(c^2-v^2\right)\frac{\partial^2 u}{\partial x^2}-2v\frac{\partial^2 u}{\partial x\,\partial t}-\frac{\partial^2 u}{\partial t^2}-b_u\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}\right)+\left(c^2-f^2\right)\frac{\left(1+\frac{\partial u}{\partial x}\right)^2\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}-\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 u}{\partial x^2}}{\left[\left(1+\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]^{3/2}}=0$$

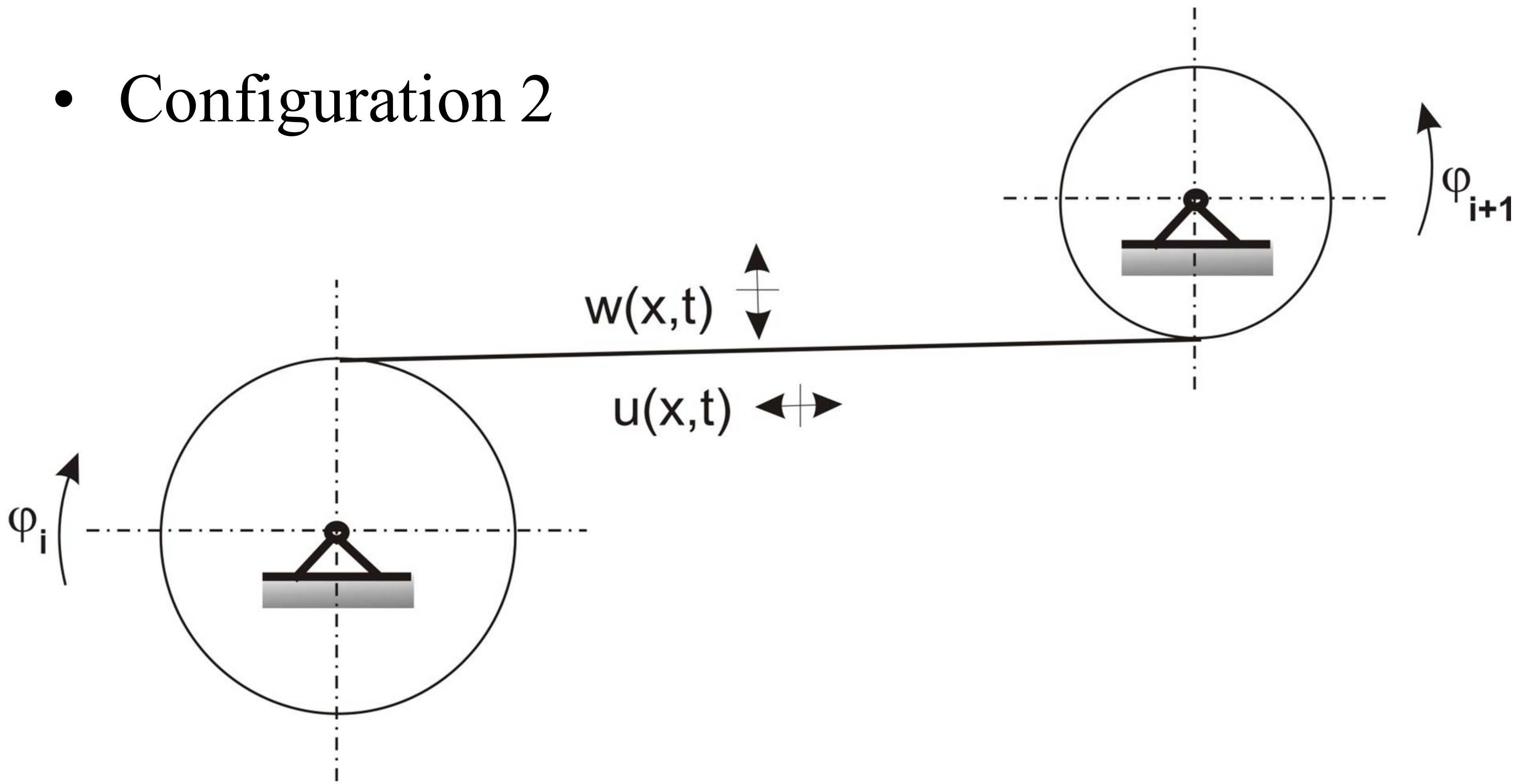
B1: Analytical – Belt-Span Configuration

- Configuration 1



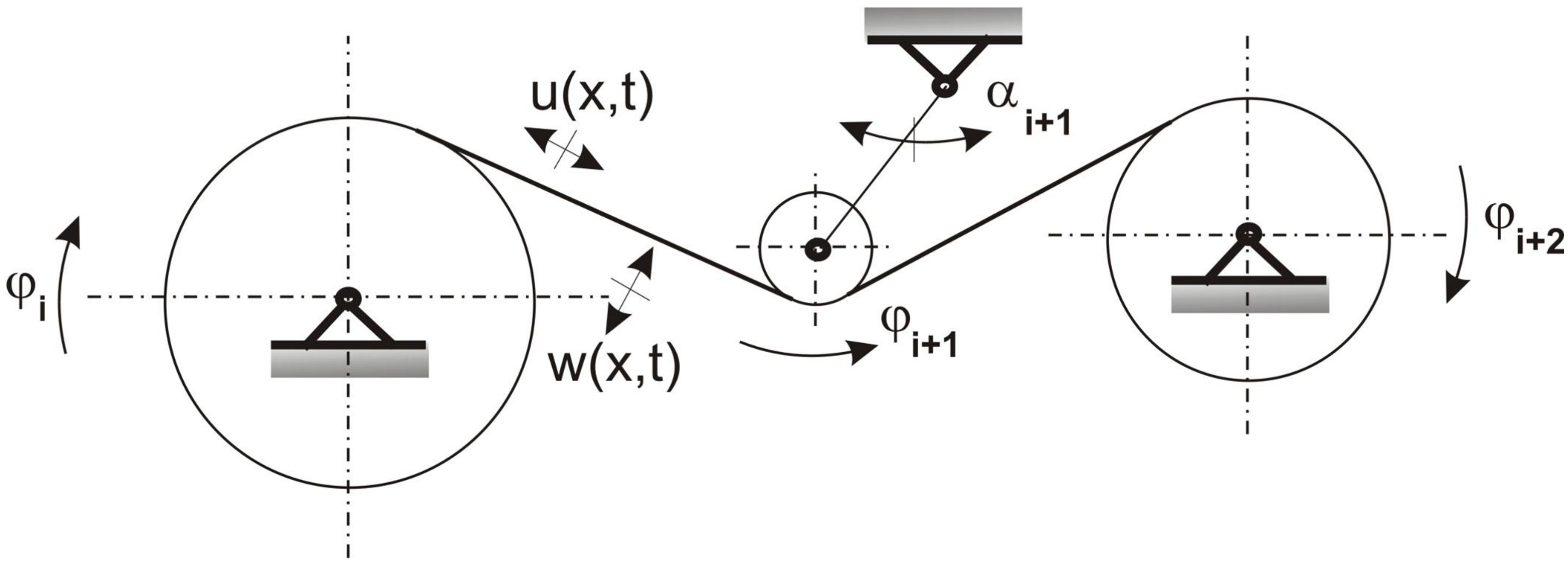
B1: Analytical – Belt-Span Configuration

- Configuration 2



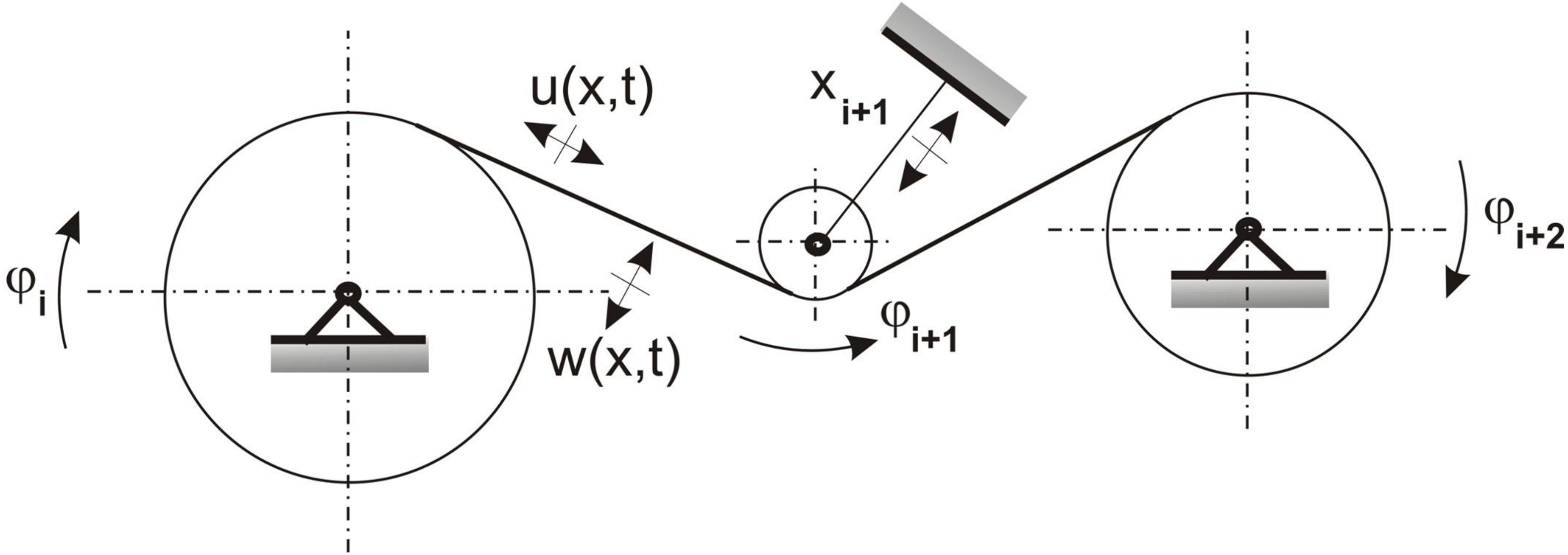
B1: Analytical – Belt-Span Configuration

- Configuration 3



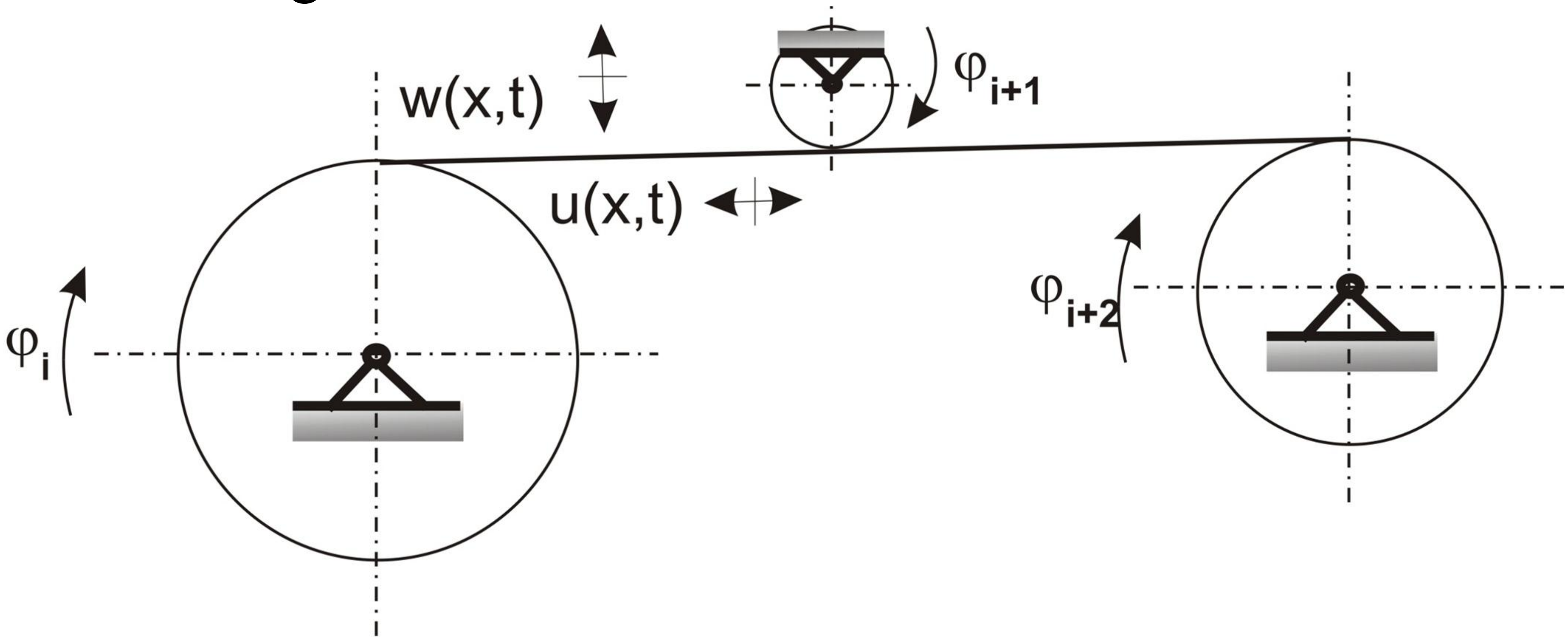
B1: Analytical – Belt-Span Configuration

- Configuration 4

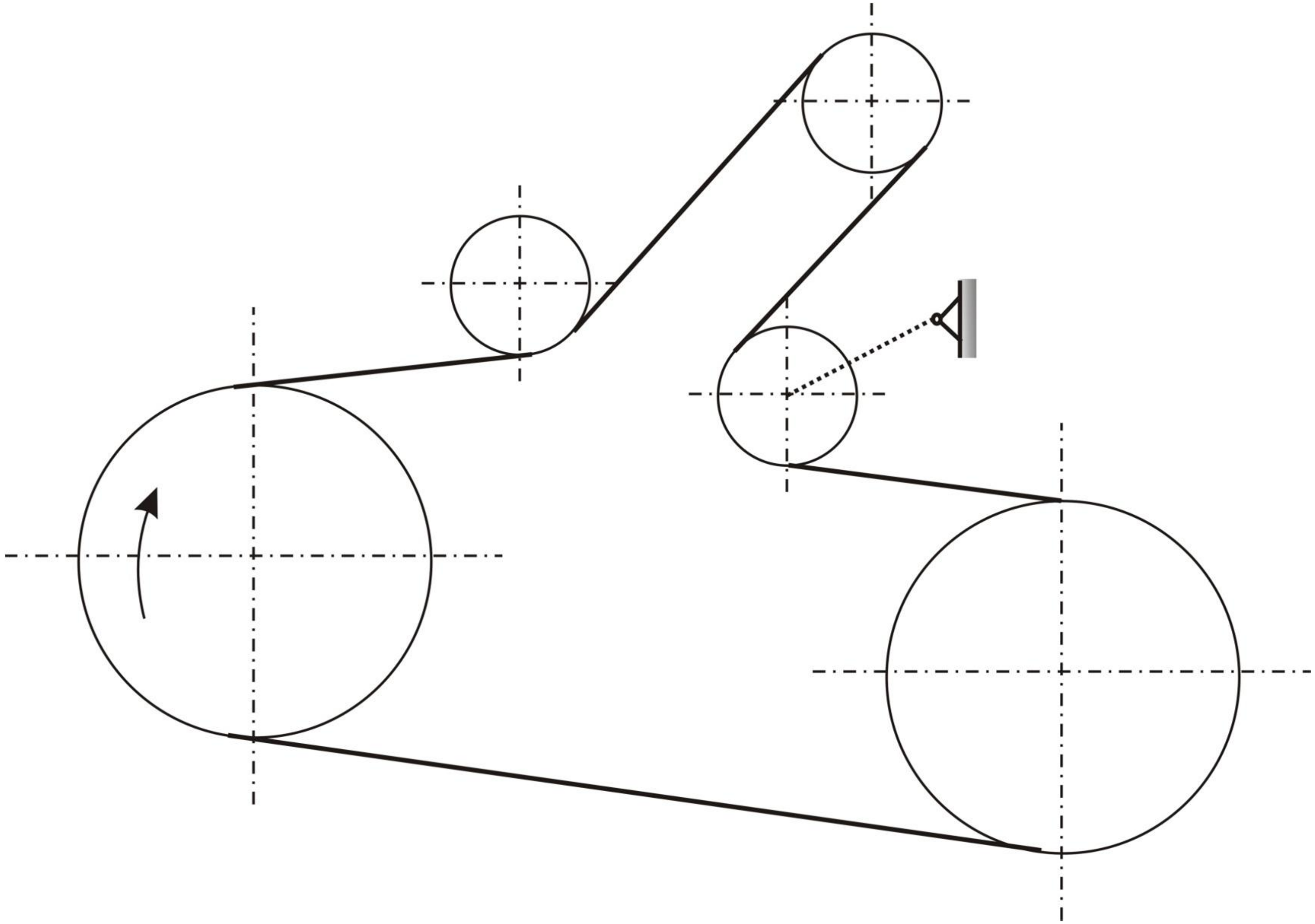


B1: Analytical – Belt-Span Configuration

- Configuration 5



B1: Analytical - Belt-Drive vibrations



C1: Results , belt span modelling (1)

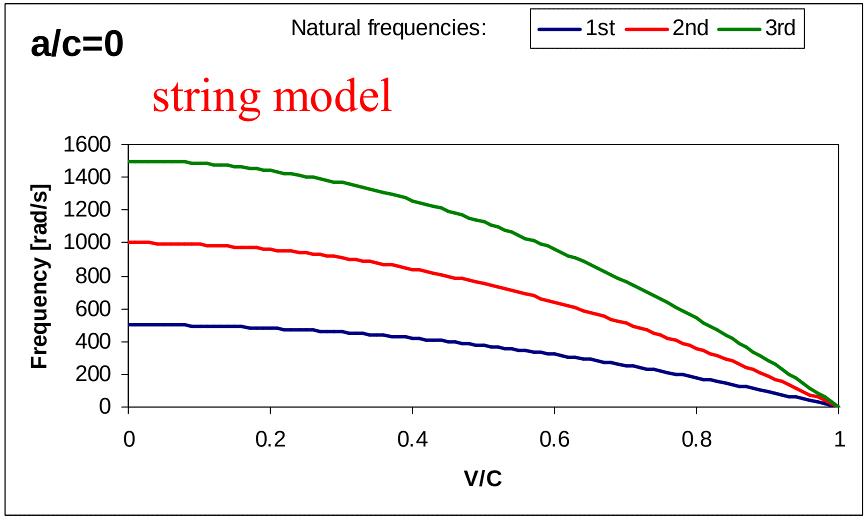
- Free transverse vibrations of the **string** model:

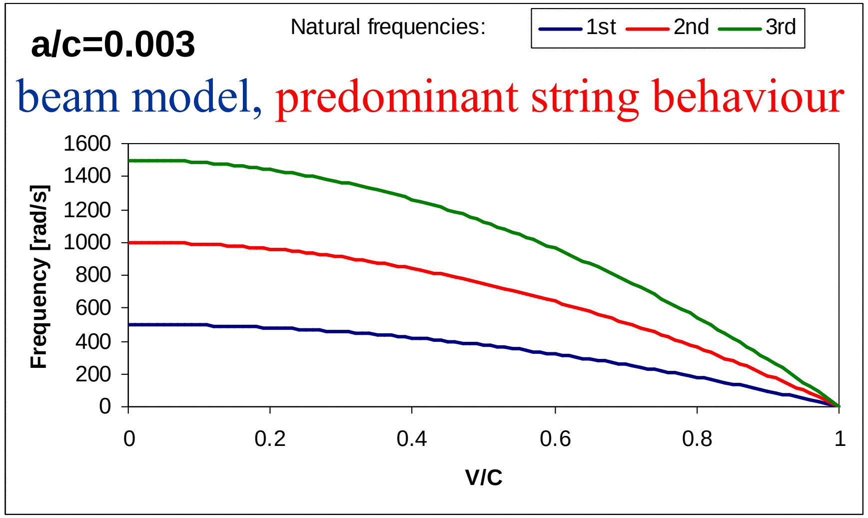
$$\left(c^2 - v^2\right)\frac{\partial^2 w}{\partial x^2} - 2v\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial^2 w}{\partial t^2} - b\left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right) = 0 \qquad c^2 = \frac{T}{\rho A}$$

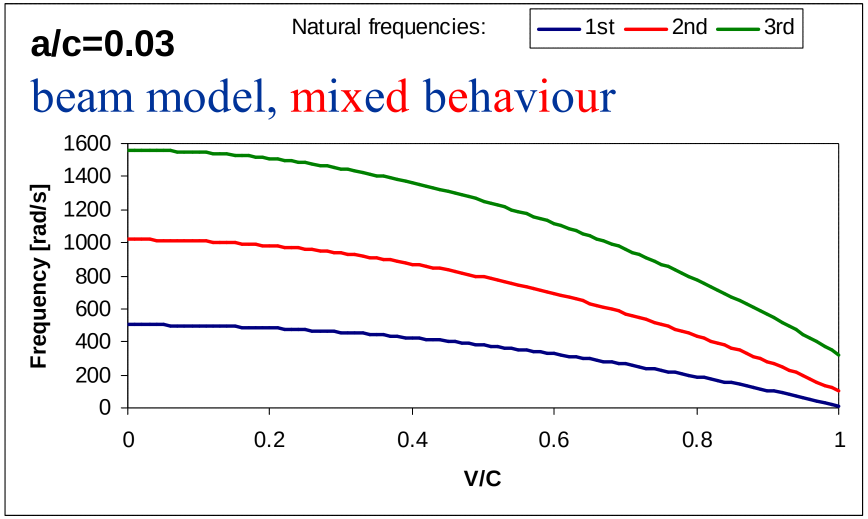
- Free transverse vibrations of the **beam** model:

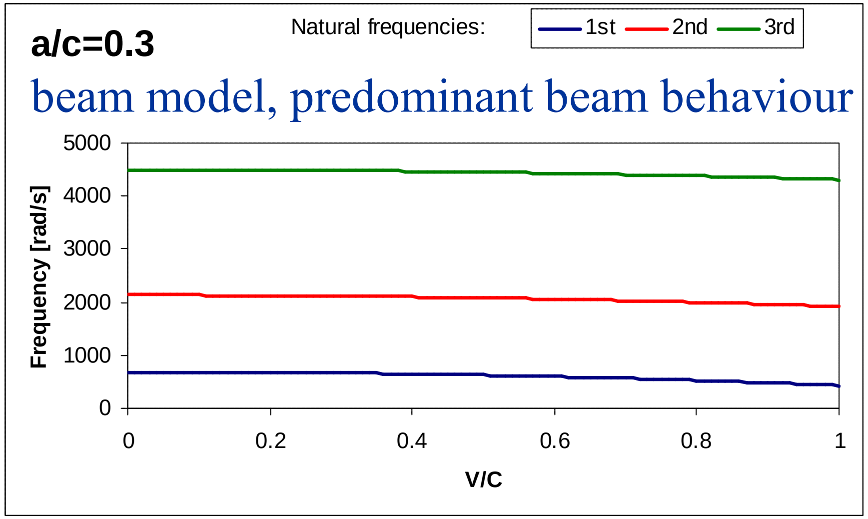
$$-a^2\frac{\partial^4 w}{\partial x^4} + \left(c^2 - v^2\right)\frac{\partial^2 w}{\partial x^2} - 2v\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial^2 w}{\partial t^2} - b\left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right) = 0 \qquad a^2 = \frac{E\,I}{\rho A}$$

C: Results, belt span modelling (2)



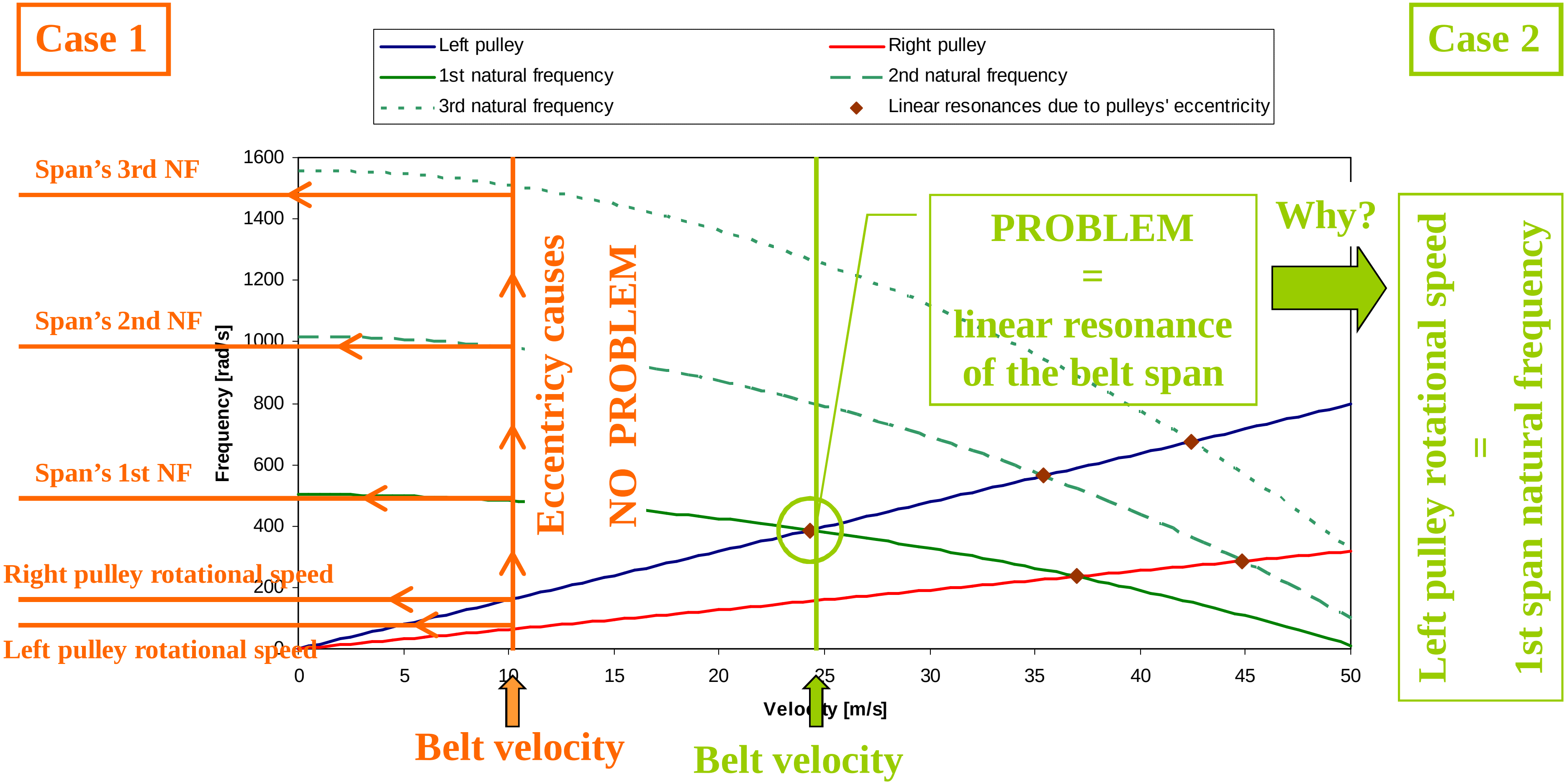






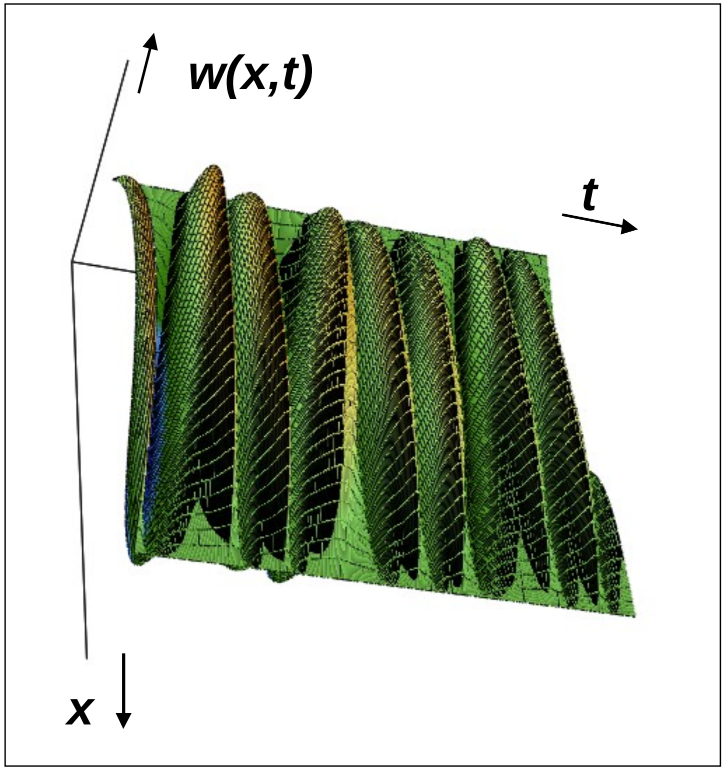
C: Results, belt span modelling (3)

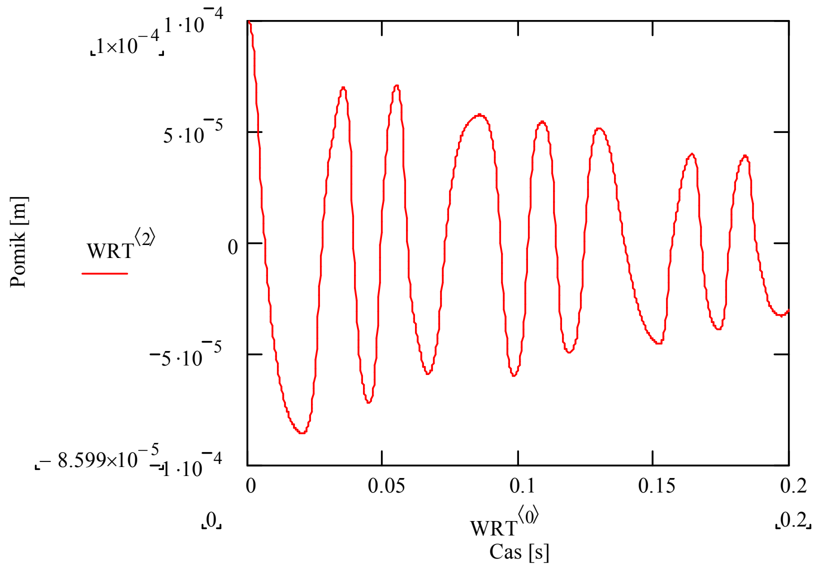
Example of usage of belt span's natural frequencies (NF)
Linear resonances due to pulleys' eccentricity



C: Results, belt span modelling (4)

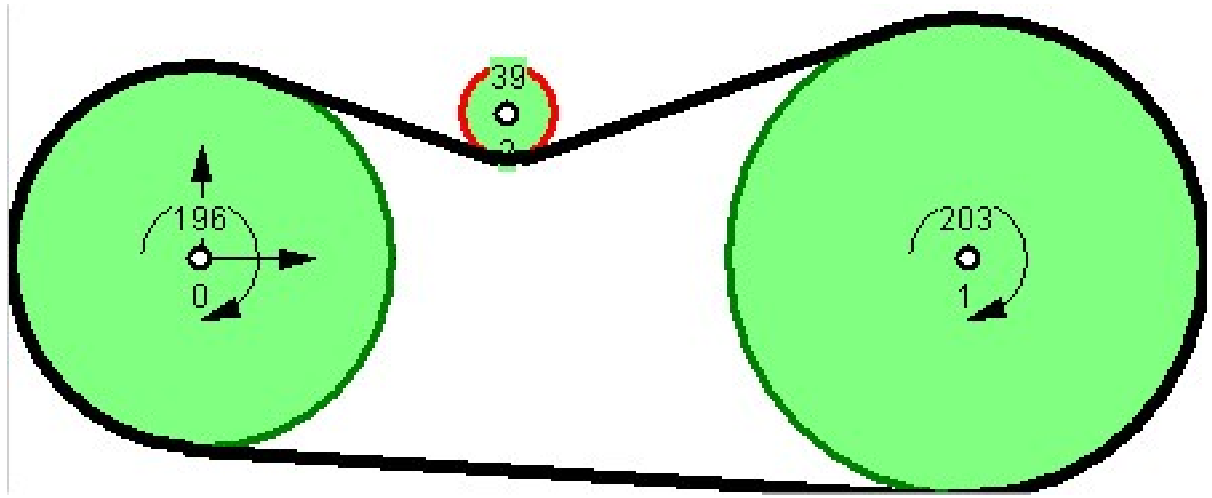
Example of the free response of the moving string – belt span





WR^T

C: Results, belt span modelling (5)
Case study – the system



C: Results, belt span modelling (6)
Case study – the data

Pulleys' data:

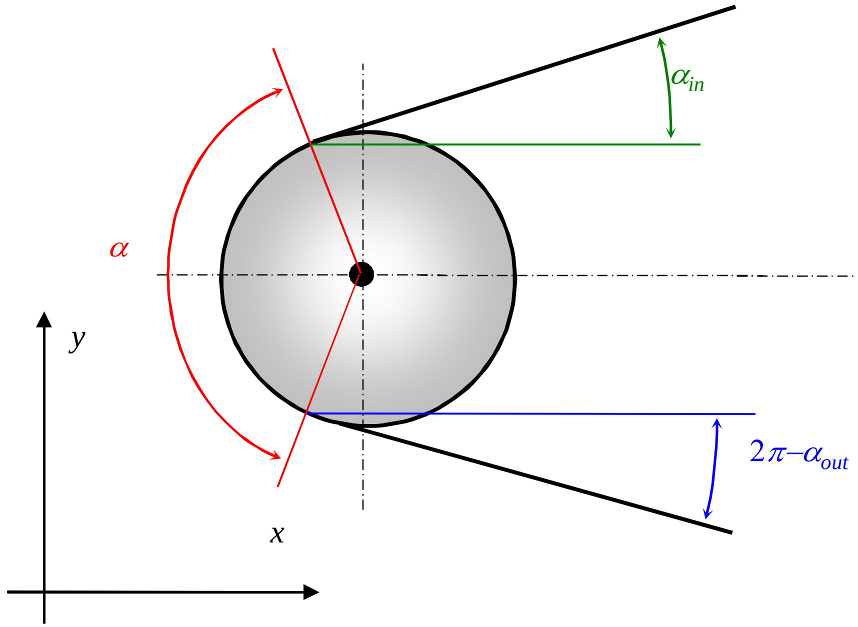
Pulley	x [m]	y [m]	r [m]	Power [W]	n [rev/min]
0	0.00	0.00	0.20	1000	1000
1	0.80	0.00	0.25	-1000	
2	0.32	0.15	0.05	0	

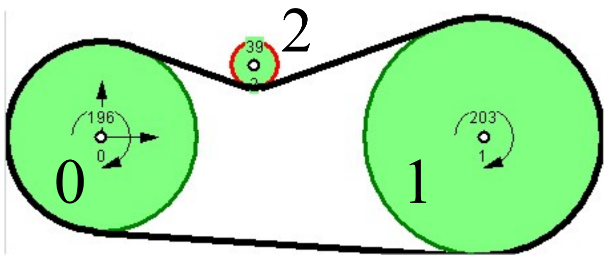
Belt's data:

μ [kg/m]	E [Pa]	A [m^2]	I [m^4]	Direction
0.015	$800 \cdot 10^6$	$180 \cdot 10^{-6}$	$896 \cdot 10^{-12}$	CW

C: Results, belt span modelling (7) Case study – basic geometry

Pulley	α [°]	α_{in} [°]	α_{out} [°]
0	196.3	70.1	266.4
1	202.9	266.4	289.3
2	39.2	289.3	70.1

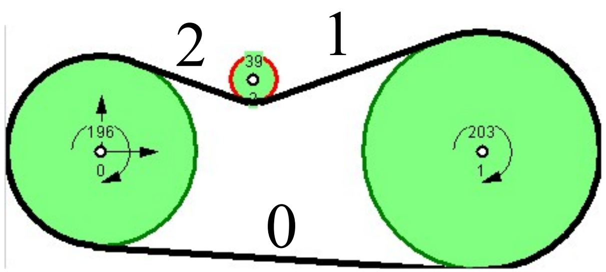


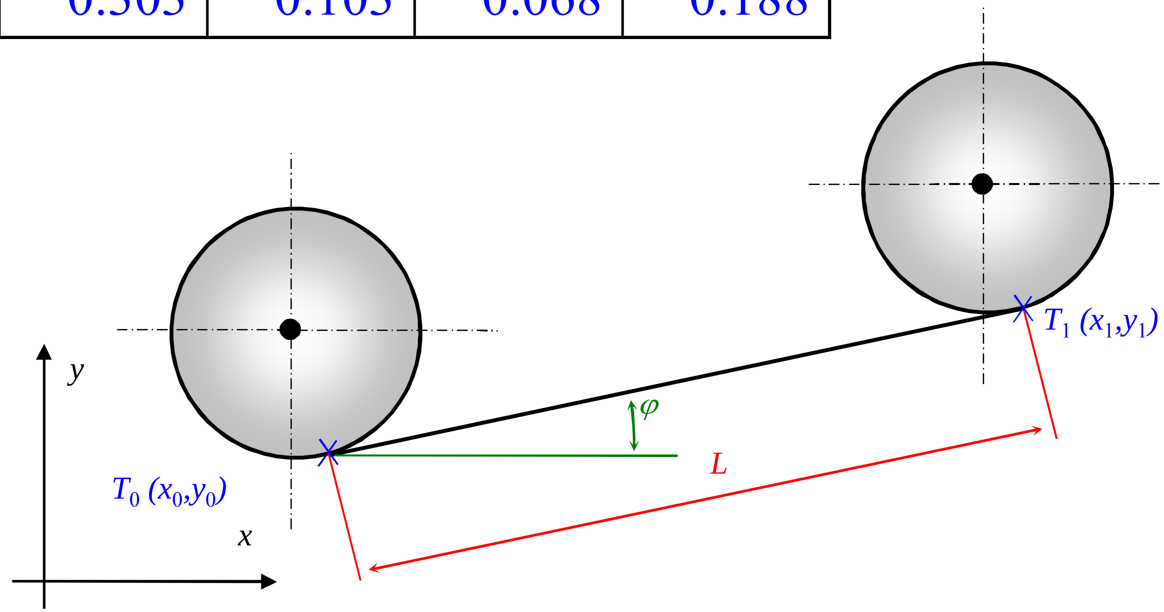


C: Results, belt span modelling (8)
Case study – basic geometry

Span	L [m]	φ [°]	x_0 [m]	y_0 [m]	x_1 [m]	y_1 [m]
0	0.798	-3.6	-0.013	-0.200	0.784	-0.250
1	0.404	-160.7	0.717	0.236	0.337	0.103
2	0.250	160.1	0.303	0.103	0.068	0.188

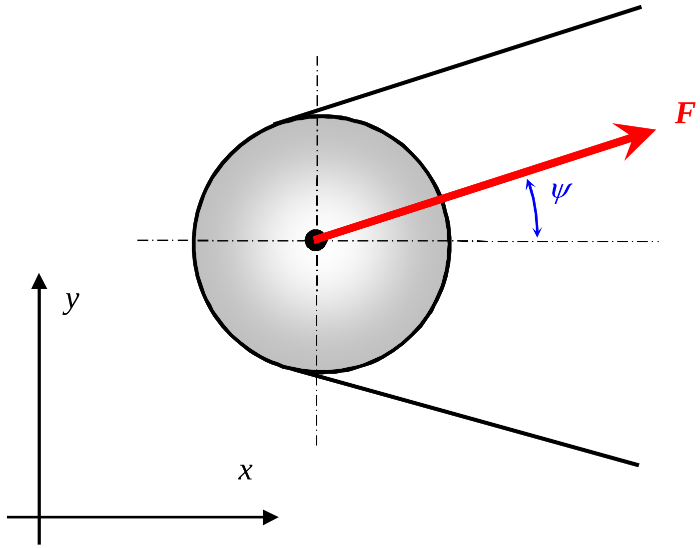
Belt length: 3.056 m

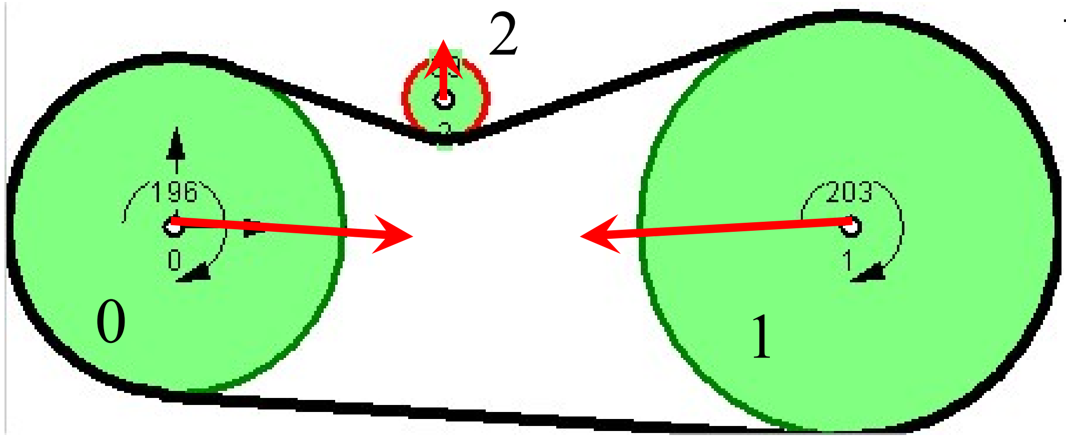




C: Results, belt span modelling (9)
Case study – hub loads

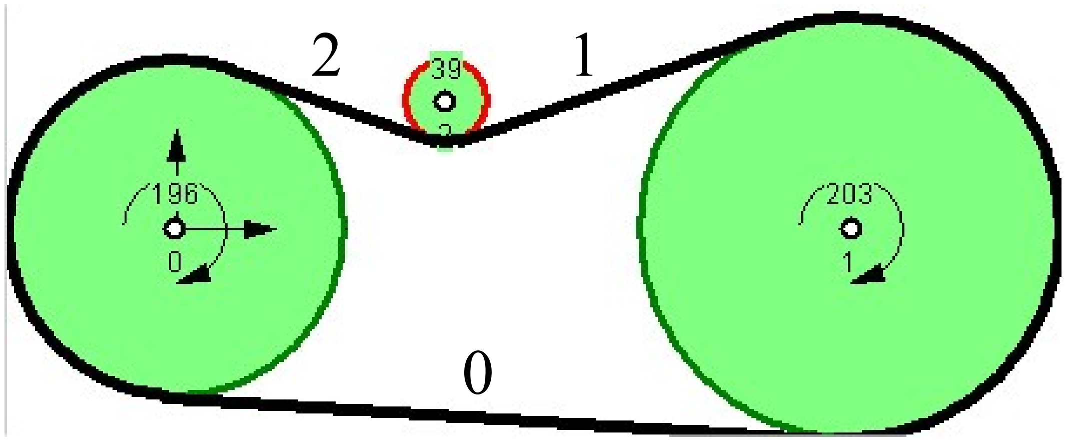
Pulley	F [N]	ψ [°]
0	101.3	-7.9
1	100.5	-177.6
2	18.2	89.7



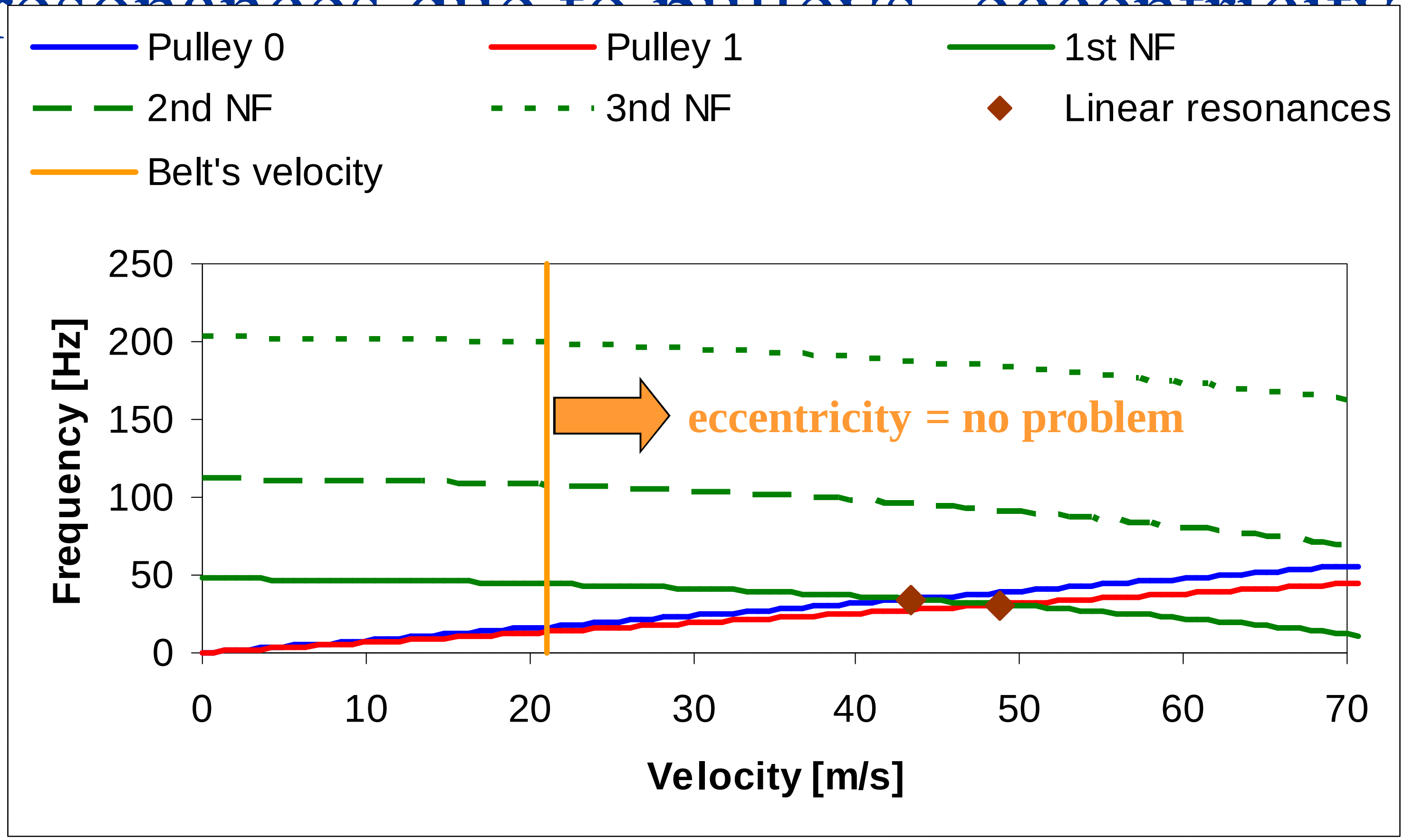


C: Results, belt span modelling (10)
Case study – span tension

Span	*F* [N]
0	74.9
1	27.2
2	27.2

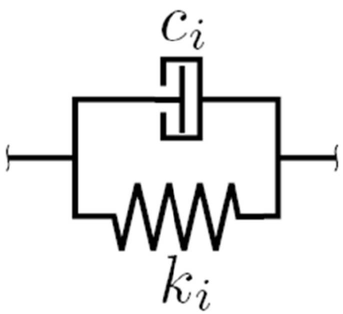


C: Results, belt span modelling (11)
Case study – natural frequencies and linear resonances due to pulleys' eccentricity



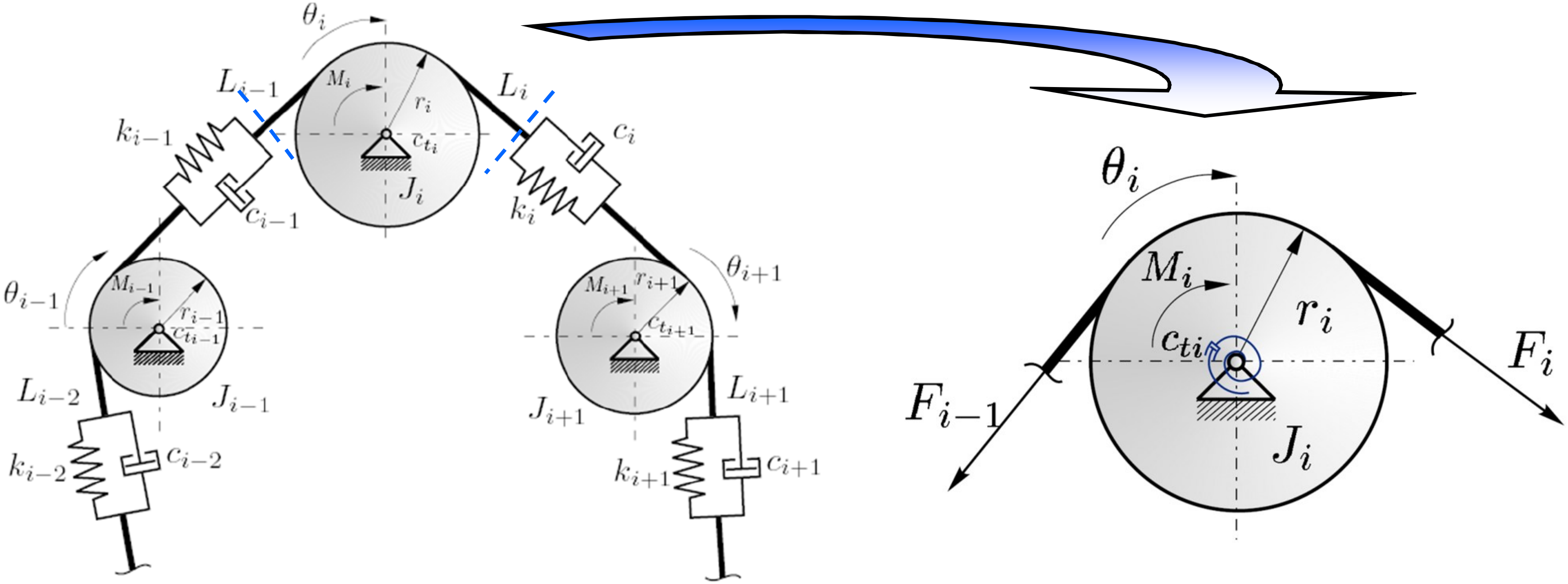
C: Results, belt drive modeling (1) Assumptions:

- There is no slip between belt and pulley
- The mass of the belt is ignored
- Fixing flexibility of pulleys is negligible
- The bending stiffness of the belt is negligible
- Model of belt span ($k_i = \dfrac{AE}{L_i}$)



C: Results, belt drive modeling (2) Pulleys rotational vibrations:

- Model with fixed pulleys (linear model)

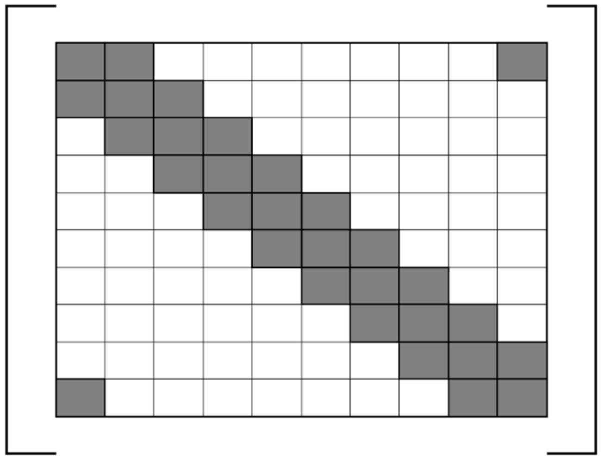


$$[J]\{\ddot{\theta}(t)\}+[C]\{\dot{\theta}(t)\}+[K]\{\theta(t)\}=\{M(t)\}$$

C: Results, belt drive modeling (3) Pulleys rotational vibrations:

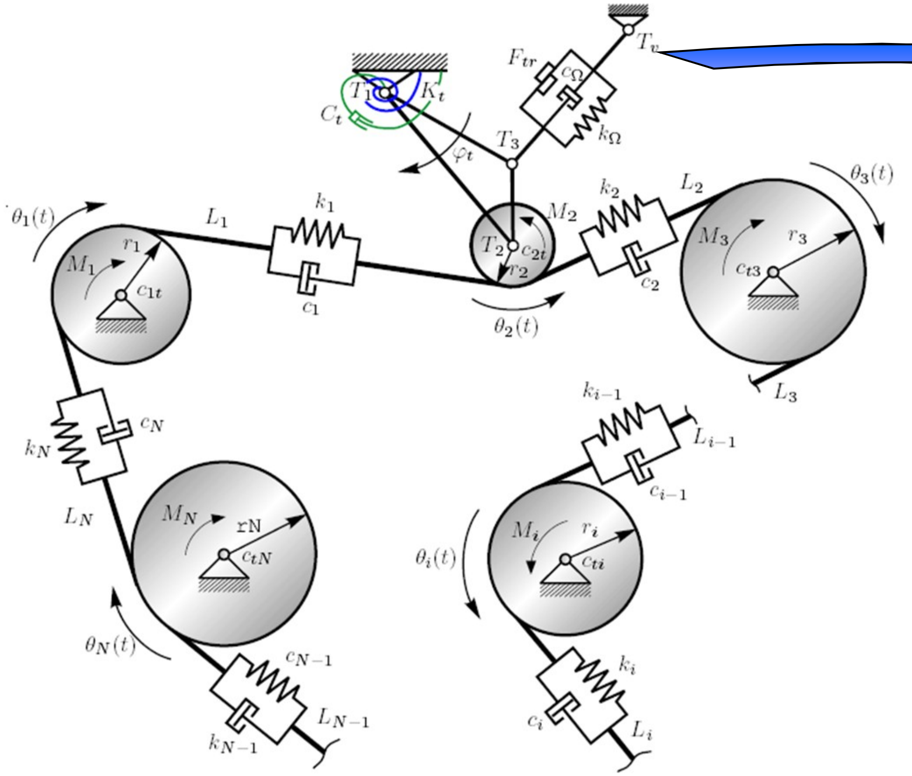
- Mass matrix $[M] = \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_N \end{bmatrix}$
- The form of matrices [K] and [C]

$[K], [C] =$

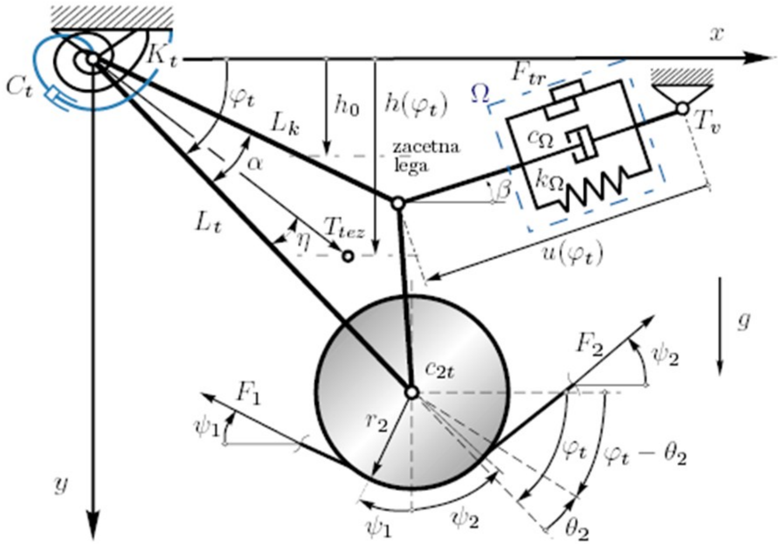


C: Results, belt drive modeling (4) Pulleys rotational vibrations:

- Model with tensioner (nonlinear model)

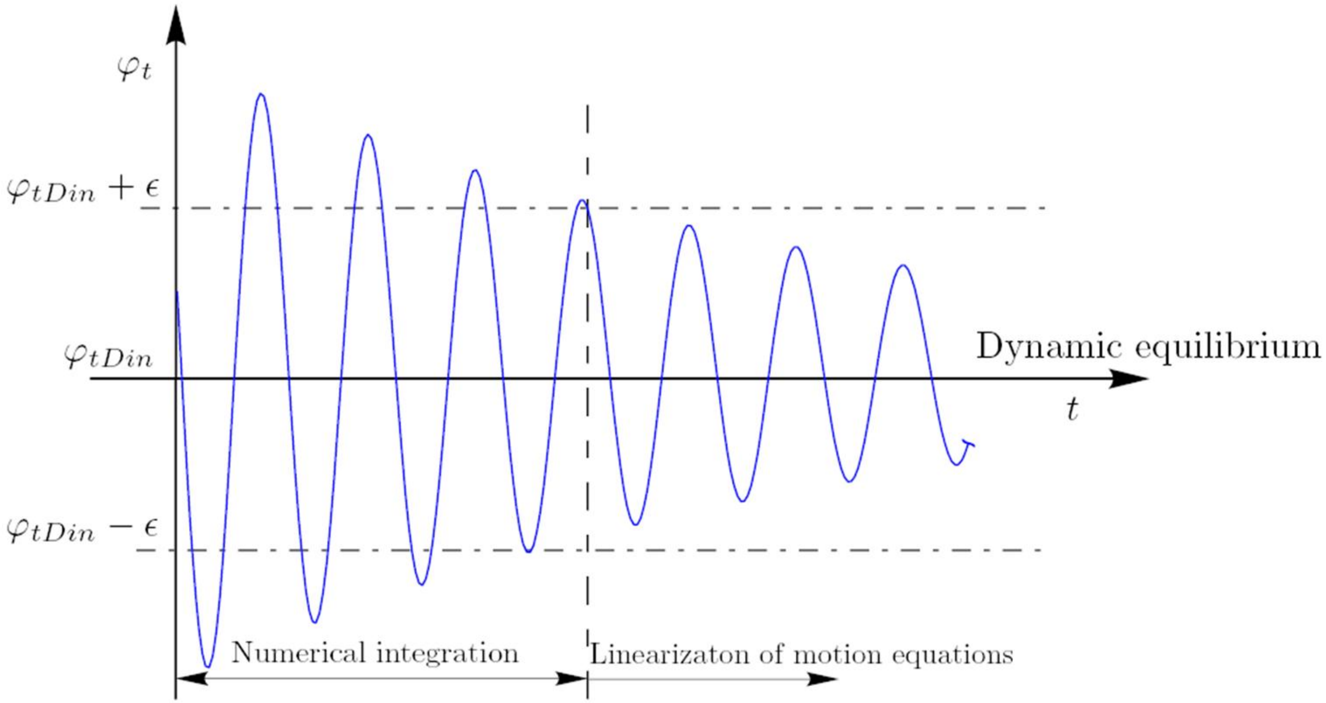


Model of tensioner

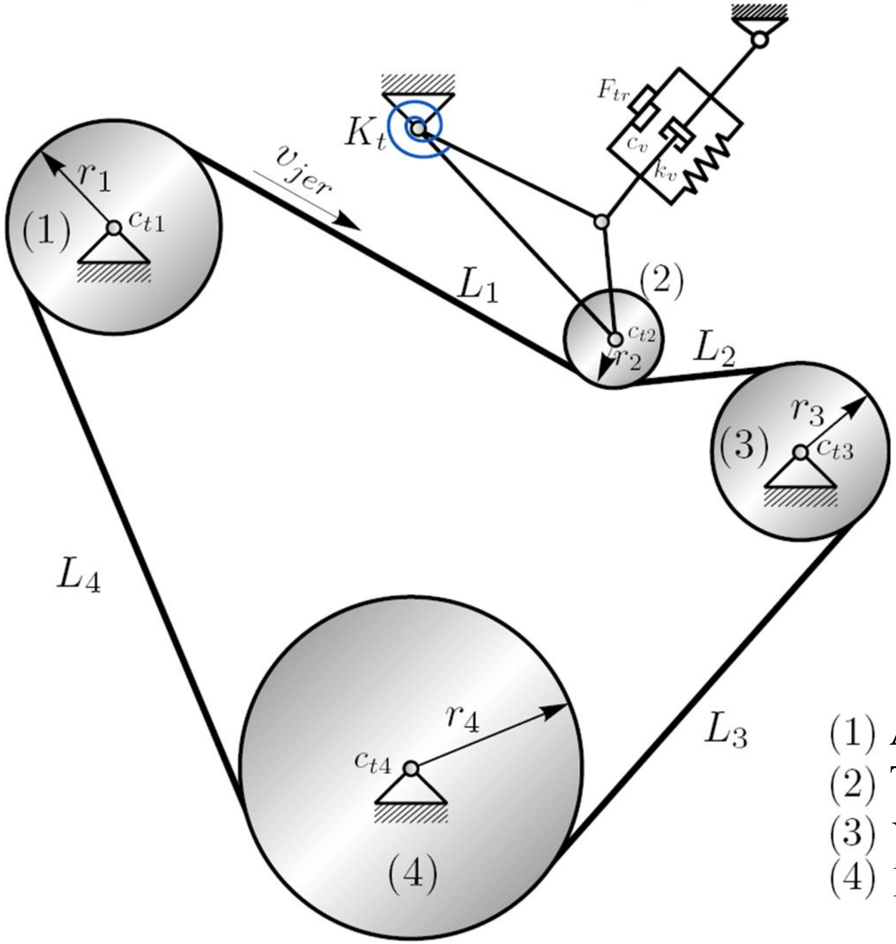


C: Results, belt drive modeling (5) Pulleys rotational vibrations:

- Linearization equations of motion around dynamic equilibrium



C: Results, belt drive modeling (6)
Case study – torsional vibrations



(1) Alternator
(2) Tensioner
(3) Ventilator
(4) Drive pulley

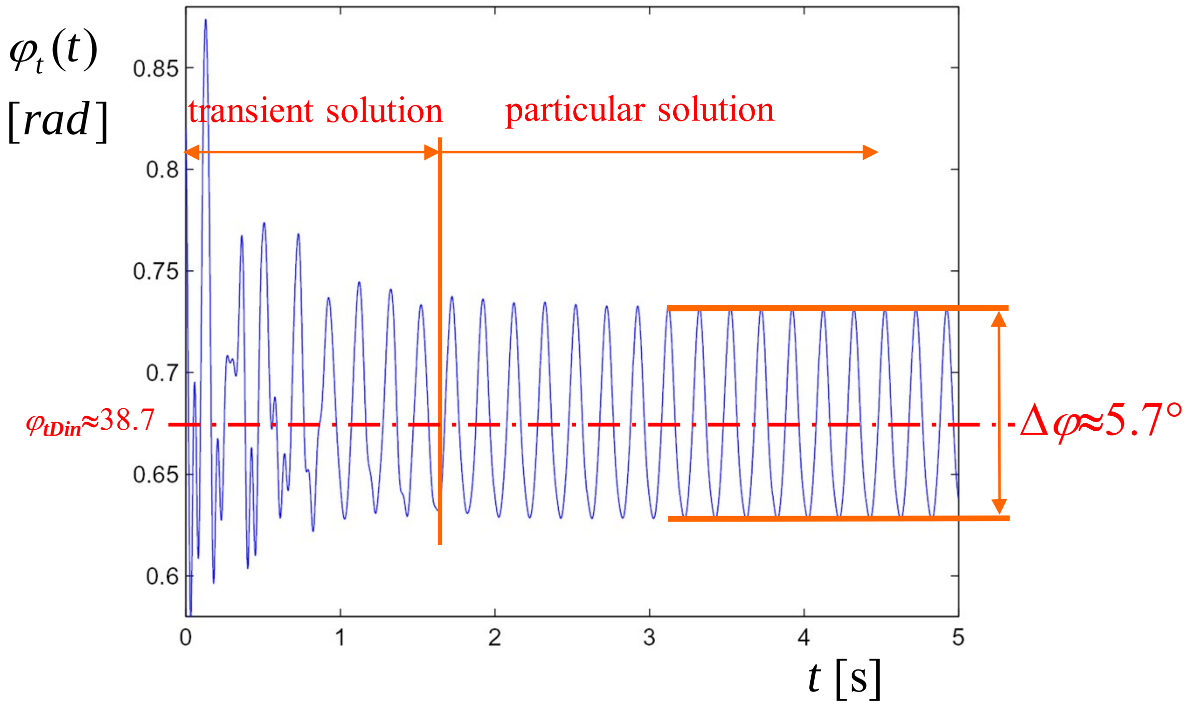
Basic technical data

C_t	0.1 Nms/rad
K_t	50 Nm/rad
c_Ω	0.1 Ns/m
k_Ω	500 N/m
A	10^{-4} m^2
E	$8 \cdot 10^6$ N/m^2
F_0	300 N
c_1, c_2, c_3, c_4	0.1 Ns/m
L_1	0.272 m
L_1	0.085 m
L_1	0.281 m
L_1	0.365 m

C: Results, belt drive modeling (7)
Case study – tensioner arm

- Kinematic exitation $\dot{\theta}_4(t) = 60(1 - e^{-t}) + 50\sin(\pi t)$

Tensioner arm oscillations



Natural frequencies:

ω_{1D}=0 rad/s

ω_{2D}=66 rad/s

ω_{3D}=68 rad/s

ω_{4D}=77 rad/s

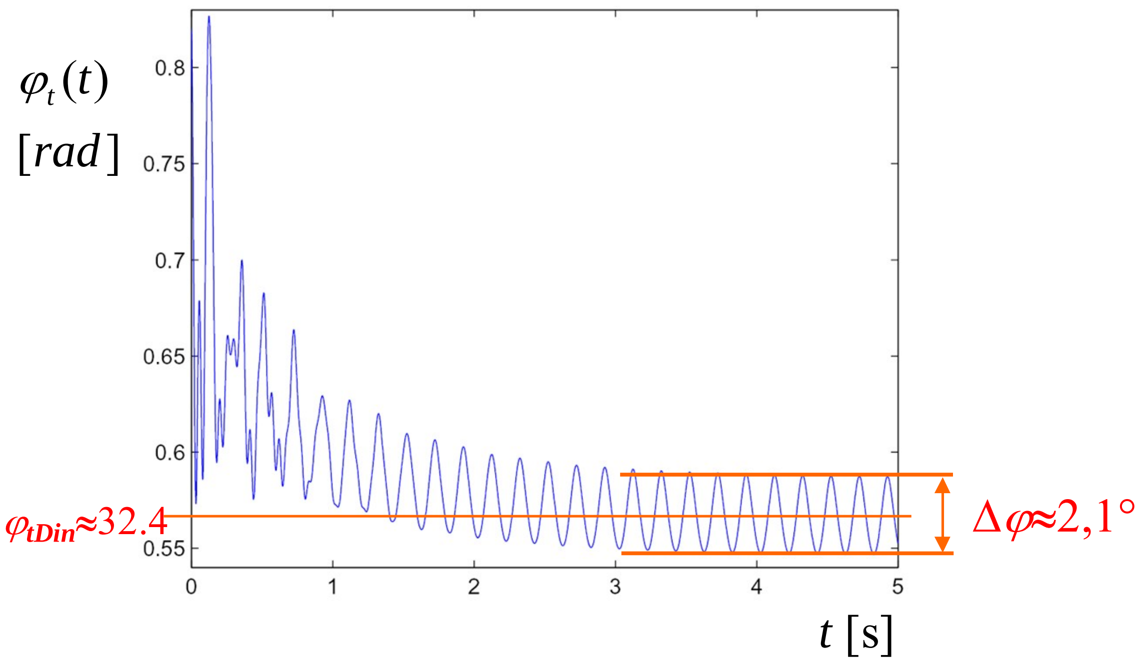
ω_{5D}=272 rad/s

System operating in region around natural frequencies

C: Results, belt drive modeling (8) Case study – Tensioner arm

- Kinematic exitation $\dot{\theta}_4(t) = 500(1 - e^{-t}) + 50\sin(\pi t)$

Tensioner arm oscillations



Natural frequencies:

ω_{1D}=0 rad/s

ω_{2D}=66 rad/s

ω_{3D}=69 rad/s

ω_{4D}=77 rad/s

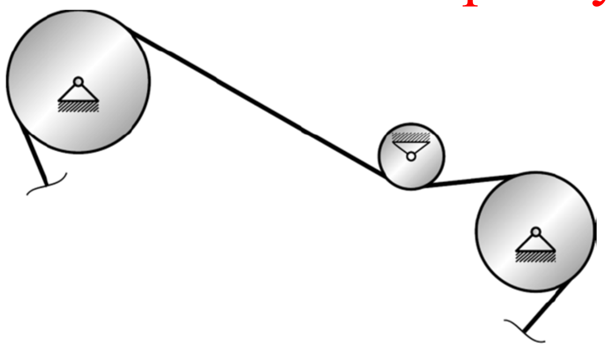
ω_{5D}=268 rad/s

System operating above natural frequencies

C: Results, belt drive modeling (9) Case study – Natural frequencies

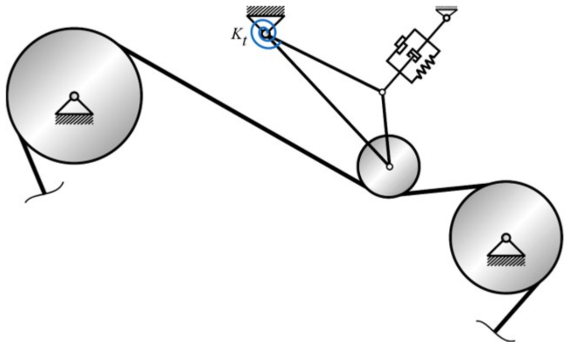
Dynamic equilibrium (Tensioner arm) φ_{tDin}=32.4°

•Model with fixed pulleys



ω_{1D} [rad/s]	ω_{3D} [rad/s]	ω_{4D} [rad/s]	ω_{5D} [rad/s]
0	65.52	76.27	276.75

•Model with tensioner



K_t [Nm/rad]	ω_{1D} [rad/s]	ω_{2D} [rad/s]	ω_{3D} [rad/s]	ω_{4D} [rad/s]	ω_{5D} [rad/s]
50	0	64.47	65.42	76.93	268.97
1000	0	66.95	73.61	79.73	268.77
5000	0	65.81	76.14	118.60	267.75
◎	0	65.52	76.27	276.75	◎

C: Results, conclusion (1)

- Preliminary literature survey concerning belts was done.
- State-of-the-art for belts is defined.
- Possible courses of action concerning belt-drive analysis were identified.

C: Results, conclusion (2)

- The analytical and semi-analytical results have been obtained for linear partial differential equations of the axially moving string and beam model.
- The string model has been chosen to start the modeling of the belt-drive's belt-spans with.

C: Results, conclusion (3)

- Research phase accomplished:
- Basic belt drive geometry.
  - Hub loads.
  - Belt-span natural frequencies (string and beam model).
  - Linear resonances due to pulleys' eccentricity.
  - Experimental set-up for pipe transmissibility tests.

C: Results, conclusion (4)

- Research phase in progress:
- Belt-drive response with fixed pulleys (pulleys’ torsional vibrations).
- Belt-drive response with tensioner (pulleys’ torsional vibrations).
- Coupled longitudinal and transversal vibrations of the belt span.

C: Results, conclusion (5)

- Application phase in progress:
  - Solver software basically defined.
  - ASCII file in, ASCII files out.
    - Computations organized by tasks (task 0 – basic geometry, task 1 – hub loads, ...).
    - Computation is carried out in separate thread.
    - Error handlers extensively used.
    - Progress monitor used at each task.
    - Object orientated code.
    - Very basic (console like) user interface.
    - Multi lingual messaging capabilities.

C: Results, conclusion (6)

- **Tensioner arm introduces geometry nonlinearities into the system.**
- Additional natural frequency (degree of freedom of tensioner arm) is very much related with tensioner spring constant $K_{t.}$
- Natural frequencies are real or complex numbers.
- In case of belt drive model with tensioner the calculation of natural frequencies and mode shapes is not possible with use of modal analysis.
- Firs natural frequency is always equal zero.

C: Results, conclusion (7)

- Research phase accomplished:

-Belt-drive response with fixed pulleys (pulleys' torsional vibrations).

  - Belt-drive response with tensioner (pulleys' torsional vibrations).

C: Results, conclusion (9)

- WHAT TO DO NEXT– major issues:
  - ⇨ Mechanics of the dynamical contact between belt and pulley for different belt designs.
  - ⇨ Linking together models of the belt drive with tensioner and belt span into one coherent model.
  - ⇨ Numerical integration of the model's response.
  - ⇨ Parameter identification of the model(s).
  - ⇨ Model verification.
  - ⇨ Upgrading the model with viscoelasticity features (if necessary).

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