

Chair of Mechanics Laboratory for Dynamics of Machines and Structures



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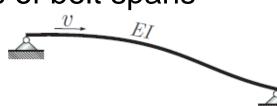
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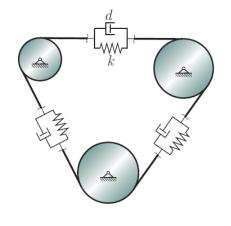
- Introduction
- Short overview of belt dynamic modelling
- Objectives
- Dynamics of belt drive model
- Contact between belt and pulley
- Numerical examples
- Conclusions

Review of (our own) belt drive modeling

Belt drive rotational vibrations



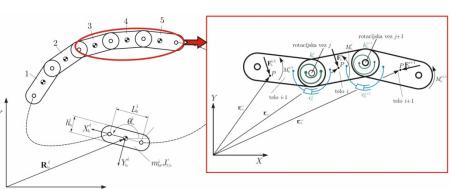




Modeling of belt drive mechanics using 3-D finite elements

Belt drive modeling using multibody system

dynamics

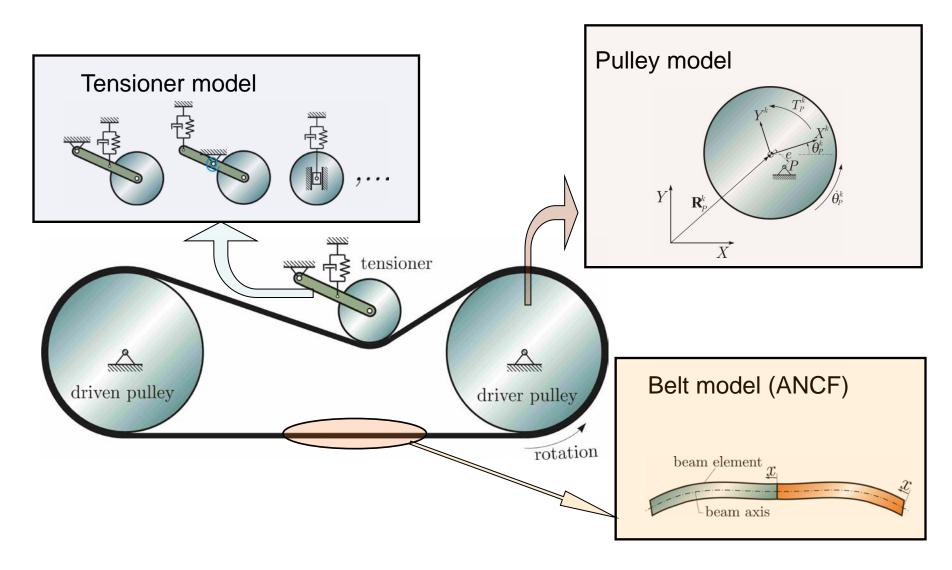


Objectives

- To develop an efficient and accurate contact model between flexible bodies
- To apply the discontinuous Coulomb friction law to the contact between belt and pulley
- To predict the belt drive dynamics at various operational conditions, in non-steady motion
- To model belt deformations exactly at arbitrary rigid body motion including the belt elastic and dissipative properties



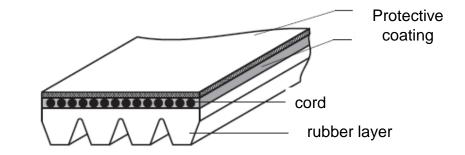
Belt drive model





Belt model (1)

Flat and poly-V belts



Viscoelastic belt material

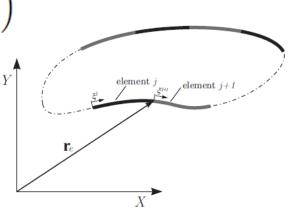
$$\sigma_l(x,t) = E\varepsilon_l(x,t) + \eta \frac{\partial \varepsilon_l(x,t)}{\partial t},$$

Lagrangian strain

$$\varepsilon_l = \frac{1}{2} \left(\mathbf{e}^{j^{\mathrm{T}}} \mathbf{S}_L \mathbf{e}^j - 1 \right)$$

Equations of motion

$$\left[\begin{array}{cc} \mathbf{M}_{e} & \mathbf{C}_{eB}^{\mathrm{T}} \\ \mathbf{C}_{eB} & \mathbf{0} \end{array}\right] \left\{\begin{array}{c} \ddot{\mathbf{e}} \\ \boldsymbol{\lambda}_{B} \end{array}\right\} = \left[\begin{array}{c} \mathbf{Q}_{f} + \mathbf{Q}_{eB} \\ \mathbf{Q}_{dB} \end{array}\right]$$



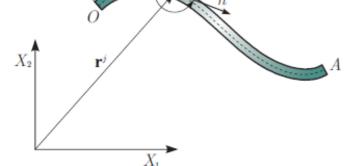


Belt model (2)

 Describing flexible elements using absolute nodal coordinate formulation (A. Shabana 1996)

$$\dot{\mathbf{r}}^{j} = \mathbf{S}\dot{\mathbf{e}}^{j}, \qquad \ddot{\mathbf{r}}^{j} = \mathbf{S}\ddot{\mathbf{e}}^{j}$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{1}(\xi) \\ \mathbf{S}_{2}(\xi) \end{bmatrix} = \begin{bmatrix} s_{1} & 0 & s_{2}L & 0 & s_{3} & 0 & s_{4}L & 0 \\ 0 & s_{1} & 0 & s_{2}L & 0 & s_{3} & 0 & s_{4}L \end{bmatrix} \xrightarrow{X_{2}}$$



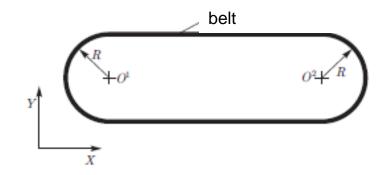
Nodal coordinates

$$e_1 = r_1|_{x=0}$$
, $e_2 = r_2|_{x=0}$, $e_5 = r_1|_{x=L}$, $e_6 = r_2|_{x=L}$
 $e_3 = \frac{\partial r_1}{\partial x}\Big|_{x=0}$, $e_4 = \frac{\partial r_2}{\partial x}\Big|_{x=0}$, $e_7 = \frac{\partial r_1}{\partial x}\Big|_{x=L}$, $e_8 = \frac{\partial r_2}{\partial x}\Big|_{x=L}$

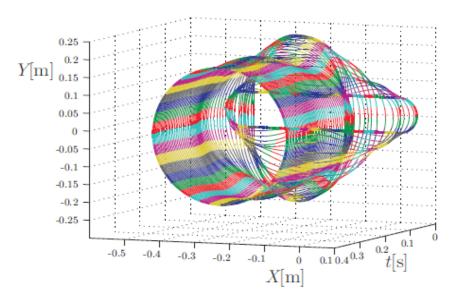


Verification of belt drive model

Initial belt configuration



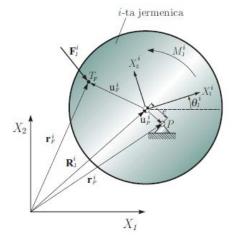
• The belt response in time





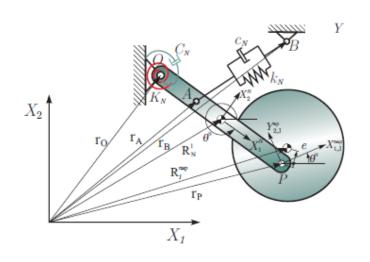
Model of pulley and tensioner

Pulley model



$$\mathbf{C}_{\mathbf{q},J} = \left[\begin{array}{cccccc} \mathbf{I} & \mathbf{A}_{\theta}^{1}\mathbf{u}_{P,J}^{1} & 0 & 0 & \dots & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{A}_{\theta}^{2}\mathbf{u}_{P,J}^{2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{I} & \mathbf{A}_{\theta}^{n_{J}}\mathbf{u}_{P,J}^{2} \\ \mathbf{0} & \dots & \dots & \theta_{J}^{pog} & \dots & \dots & 0 \end{array} \right]$$

Tensioner model



$$\mathbf{C}_{\mathbf{q},JN} = egin{bmatrix} & \vdots & & \vdots & \\ & \mathbf{C}_{q,J} & & -\left[\mathbf{I},\mathbf{A}_{ heta}^{N}\mathbf{u}_{P}^{N}
ight] & & \vdots & \\ & & & \vdots & & \\ & & & & \mathbf{C}_{q,N_{1}} & & \end{bmatrix}$$

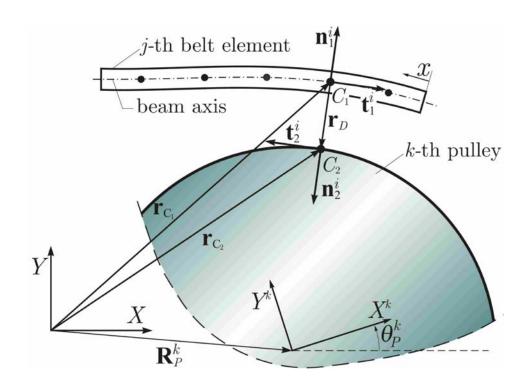


Contact formulation between belt and pulley

Contact kinematics

$$\ddot{g}_N^i = \mathbf{W}_N^{i} \ddot{\mathbf{q}}^{jk} + w_N^i$$
...
$$\mathbf{W}_N^i \ddot{\mathbf{q}}^{jk} + w_N^i$$

$$\ddot{g}_T^i = \mathbf{W}_T^{i} \ddot{\mathbf{q}}^{jk} + w_T^i$$



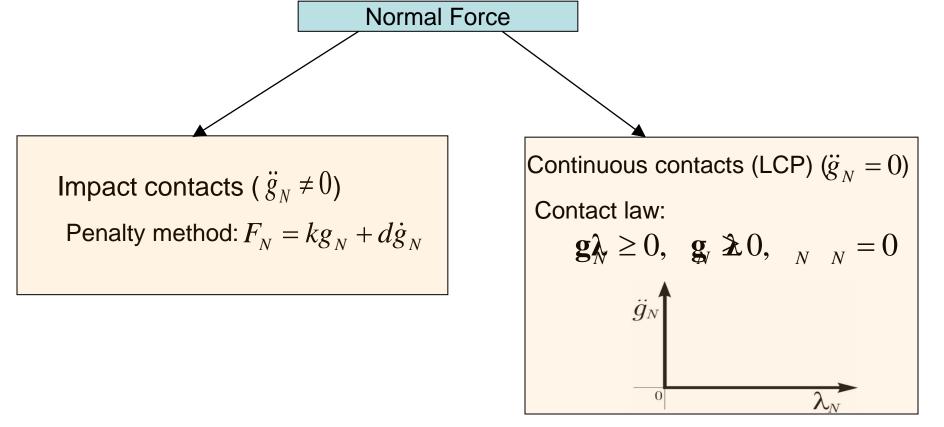
Equation of motion including contact forces

$$\ddot{\mathbf{q}}_r = \mathbf{H}_F \left(\mathbf{W}_N \boldsymbol{\lambda}_N + \mathbf{W}_T \boldsymbol{\lambda}_T \right) + \mathbf{h}$$



Normal contact force

 Formulation of contact problem as LCP together with penalty method

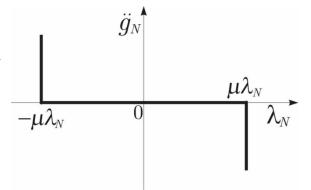




Frictional contact force

Using discontinuous Coulomb friction law

$$\begin{cases} \text{sticking:} & |\lambda_T| < \mu \lambda_N \implies |\ddot{g}_T| = 0 \\ \text{sliding:} & |\lambda_T| = \mu \lambda_N \implies |\ddot{g}_T| > 0 \end{cases}$$



 Using LCP in order to compute frictional forces of possible sticking contacts

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{b}, \quad \mathbf{y} \ge 0, \quad \mathbf{x} \ge 0, \quad \mathbf{y}^{\mathrm{T}}\mathbf{x} = 0,$$

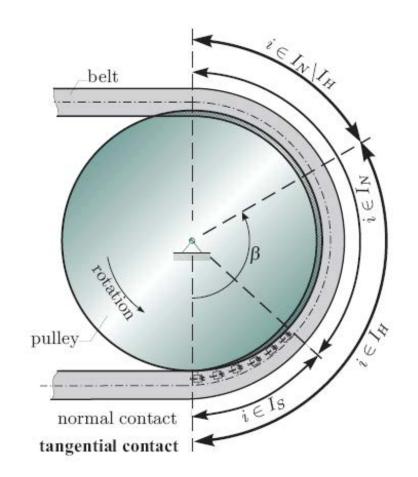
$$\mathbf{A} = \begin{bmatrix} \mathbf{W}_{NC}^{\mathrm{T}} \mathbf{T} \mathbf{H}_{F} \left[\mathbf{W}_{NC} + \mathbf{W}_{GN} \overline{\overline{\mu}}_{GN} + \mathbf{W}_{H} \overline{\overline{\mu}}_{0HN} \right] & -\mathbf{W}_{NC}^{\mathrm{T}} \mathbf{T} \mathbf{H}_{F} \mathbf{W}_{H} & \mathbf{0} \\ -\mathbf{W}_{H}^{\mathrm{T}} \mathbf{T} \mathbf{H}_{F} \left[\mathbf{W}_{NC} + \mathbf{W}_{GN} \overline{\overline{\mu}}_{GN} + \mathbf{W}_{H} \overline{\overline{\mu}}_{0HN} \right] & \mathbf{W}_{H}^{\mathrm{T}} \mathbf{T} \mathbf{H}_{F} \mathbf{W}_{H} & \mathbf{I} \\ 2 \overline{\overline{\mu}}_{0HN} & -\mathbf{I} & \mathbf{0} \end{bmatrix}$$



Capabilities of presented contact model

 The contact sets in the case of steadily operating belt drive

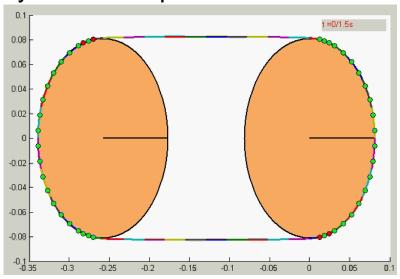
 Prediction of contact forces and the response of entire belt drive in the case of non-steady belt drive operational conditions



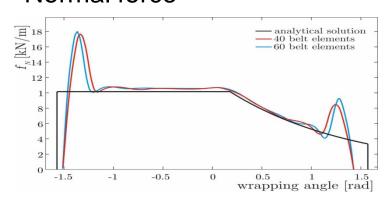


Two-pulley belt drive example (steady operation)

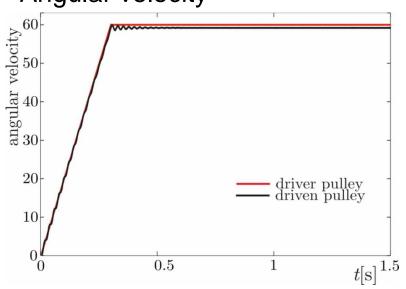
Dynamics response of belt drive



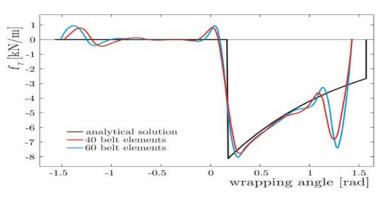
Normal force



Angular velocity



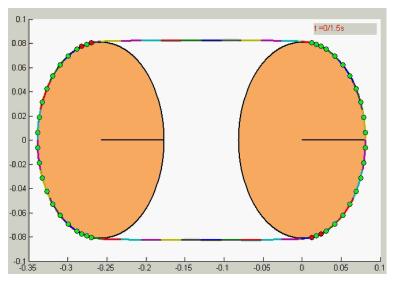
Friction force



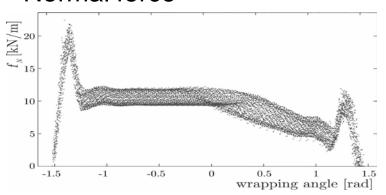


Two-pulley belt drive example (non-steady operation)

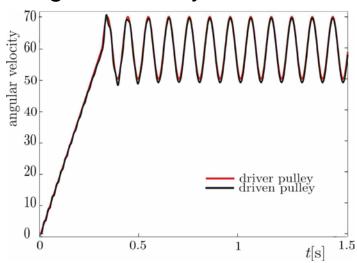
Dynamics response of belt drive



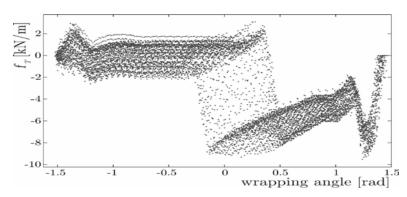
Normal force



Angular velocity



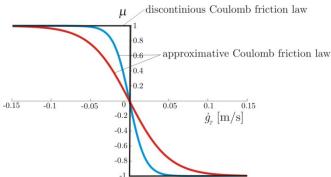
Friction force



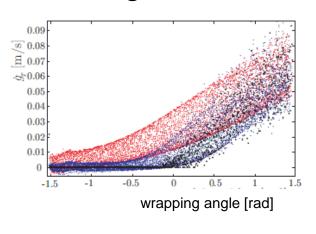


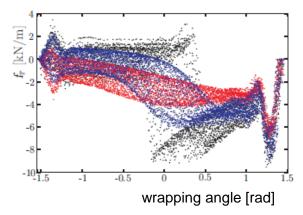
Comparison between friction laws

Approximate friction law



Relative tangential velocities and frictional forces



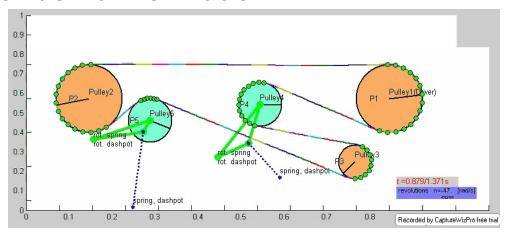


Computational time

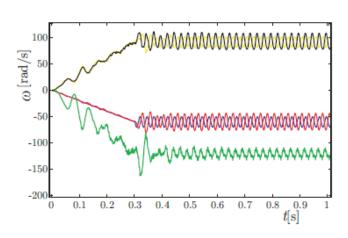
	discontinuous Coulomb	approximate friction law			
	friction law	$\nu_{s} = 10$	$\nu_s = 20$	$\nu_s = 50$	
computation time [h]	20	30,4	77,2	150	

Applicability of belt drive model

Numerical belt drive model



Angular velocities



Conclusions

- Belt was modeled as planar beam element in ANCS
- Viscoelastic material for longitudinal direction
- Rayleigh damping for transverse direction
- Belt-pulley contact was modeled with LCP including penalty method
- Comparison to analytical solutions
- Ability to predict non-steady operations of the belt drive

Thank you for your attention

