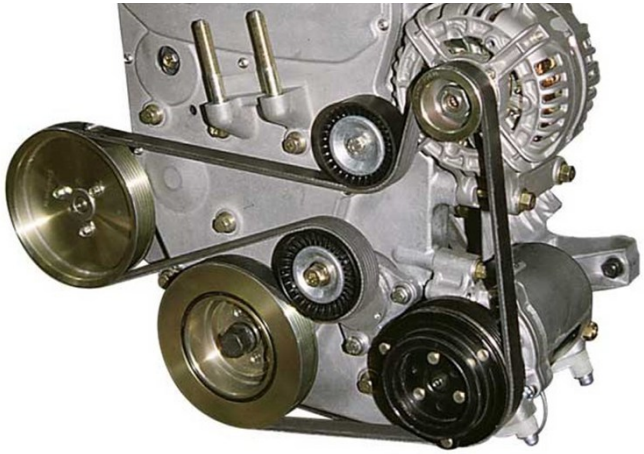
Univerza *v Ljubljani*
Fakulteta za *strojništvo*
Laboratorij za dinamiko strojev in konstrukcij

Nihanja jermenov

Dinamika strojev



Ljubljana, 6. 12. 2013

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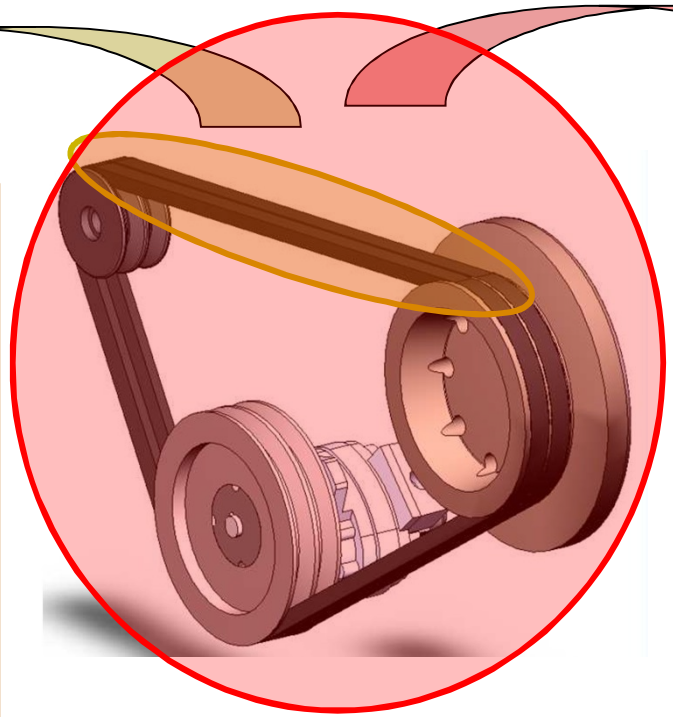
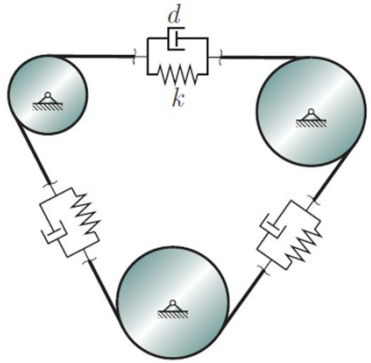
Klasifikacija nihanj jermenov (1)

Prečne vibracije veje jermena

1. Osno premikajoča se struna
2. Osno premikajoča se plošča
3. Osno premikajoč se nosilec

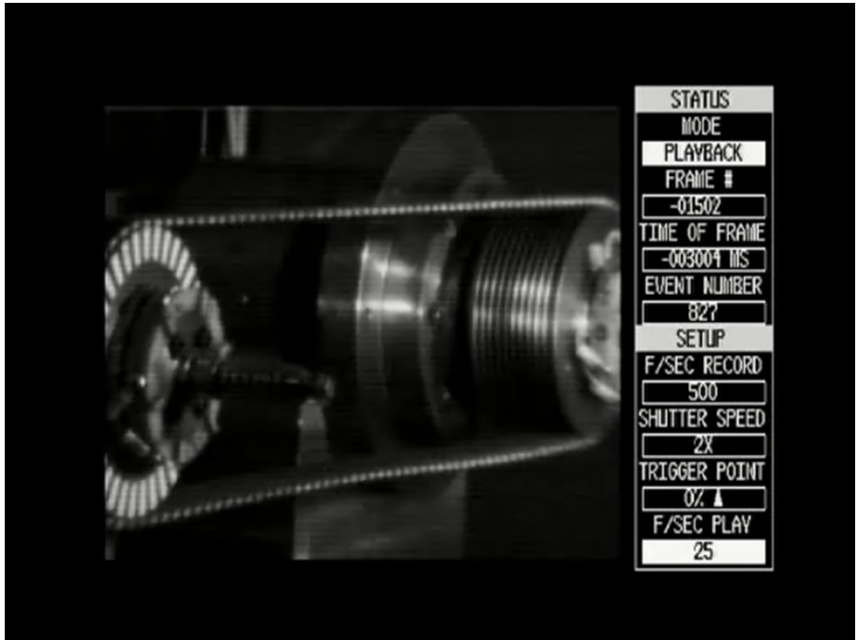


Torzijske vibracije jermenskega gonila



1. Kelvinovo telo
2. Hookovo telo
3. …

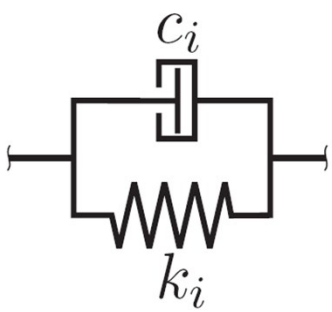
Klasifikacija nihanj jermenov (2)



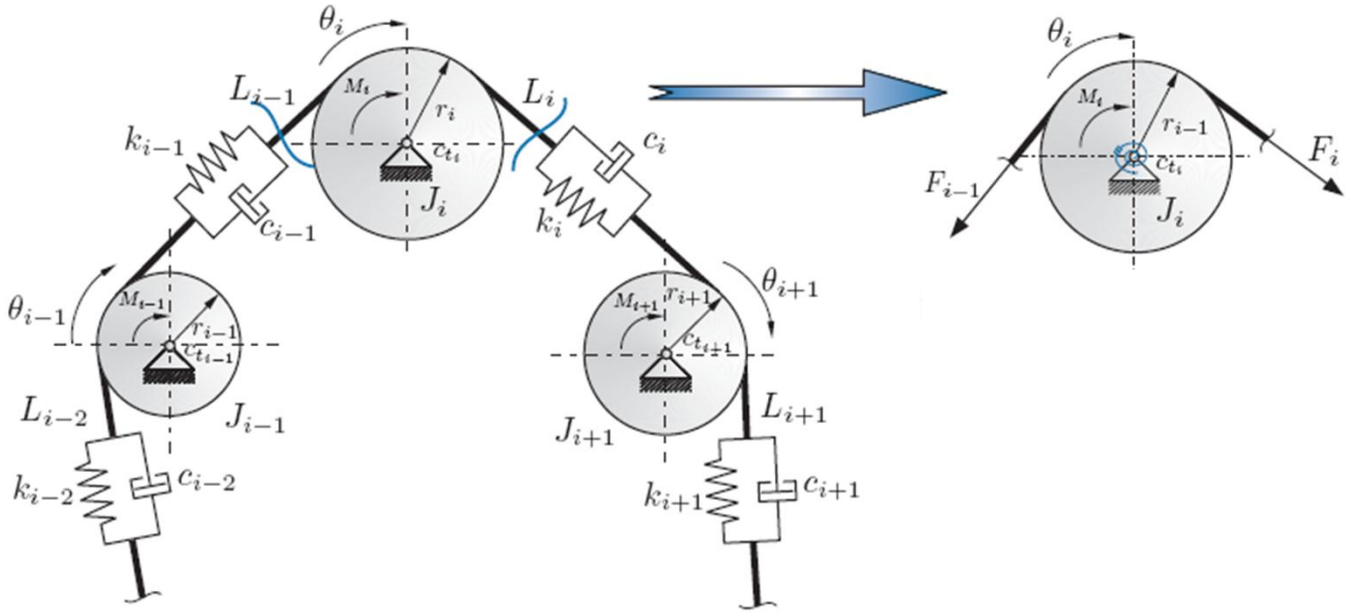
Modeliranje torzijskih nihanj jermenskega gonila

Predpostavke

- Jermensko gonilo modeliramo v okviru dinamike togih teles
- Prožnost vpetja jermenic zanemarimo
- Med jermenom in jermenico zanemarimo zdrs
- Maso jermena zanemarimo
- Upogibna togost je v primerjavi z osno togostjo zanemarljiva
- Model veje jermena ($k_i = \frac{AE}{L_i}$)



Model torzijskih nihanj jermenskega pogona (1)



$$J_i\ddot{\theta}_i(t) = \sum_r M_r^{(i)}, \quad i = 1, 2, ..., N$$

$$F_i = F_0 + c_i(r_{i+1}\dot{\theta}_{i+1}(t) - r_i\dot{\theta}_i(t)) + k_i(r_{i+1}\theta_{i+1}(t) - r_i\theta_i(t)), \quad i = 1, 2, ..., N$$

$$[J]\{\ddot{\theta}(t)\} + [C]\{\dot{\theta}(t)\} + [K]\{\theta(t)\} = \{M(t)\}$$

Model torzijskih nihanj jermenskega pogona (2)

Masna matrika:

$$[J] = \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_N \end{bmatrix}$$

Matrika dušenja:

$$[C] = \begin{bmatrix} c_{t1} + (c_N + c_1)r_1^2 & -c_1 r_1 r_2 & \cdots & -c_N r_N r_1 \\ -c_1 r_1 r_2 & c_{t2} + (c_1 + c_2)r_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -c_N r_N r_1 & 0 & \cdots & c_{tN} + (c_N + c_{N-1})r_N^2 \end{bmatrix}$$

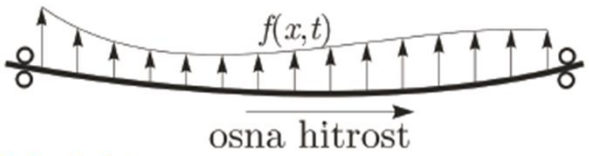
Togostna matrika:

$$[K] = \begin{bmatrix} (k_N + k_1)r_1^2 & -k_1 r_1 r_2 & \cdots & -k_N r_N r_1 \\ -k_1 r_1 r_2 & (k_1 + k_2)r_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -k_N r_N r_1 & 0 & \cdots & (k_{N-1} + k_N)r_N^2 \end{bmatrix}$$

Gibalna enačba (določitev lastnih frekvenc):

$$[J]\{\ddot{\theta}(t)\} + [C]\{\dot{\theta}(t)\} + [K]\{\theta(t)\} = \{M(t)\}$$

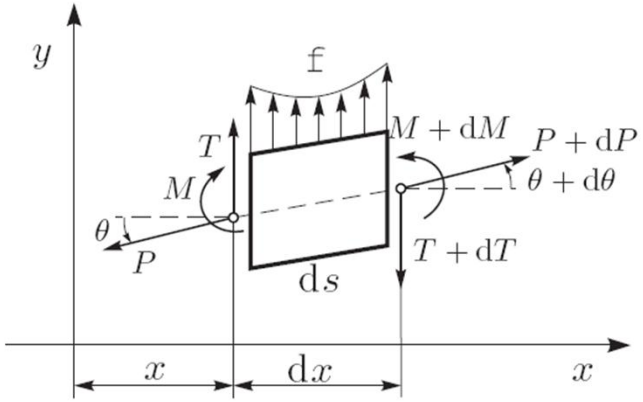
Modeliranje prečnih nihanj vej jermena

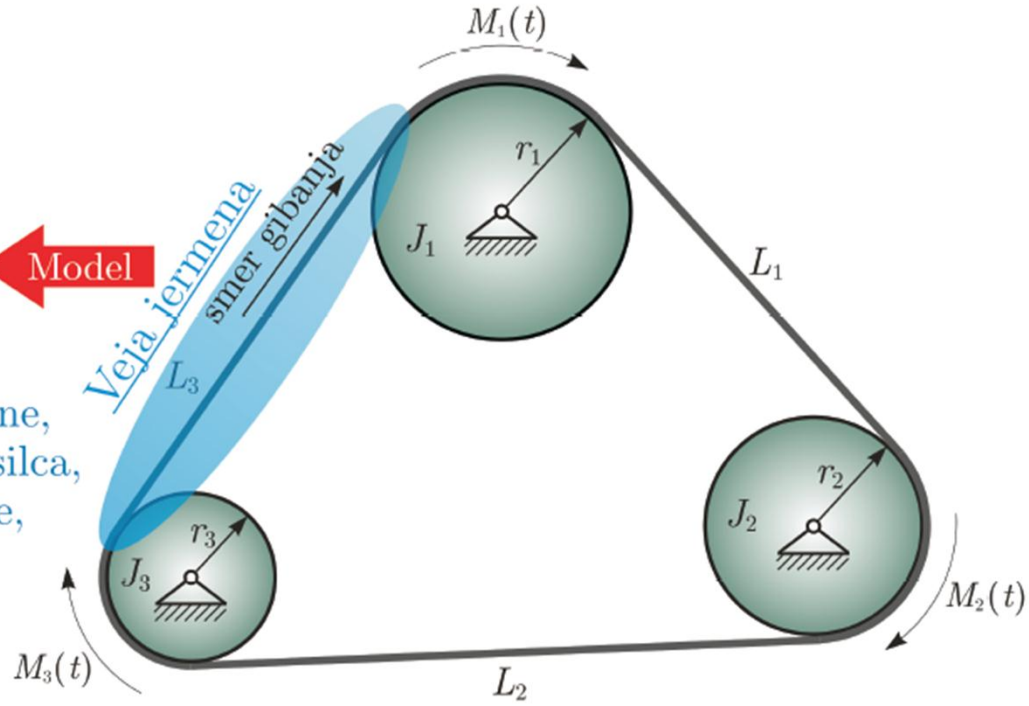


Model

Model jermena:

- model osno premikajoče se strune,
- model osno premikjočega se nosilca,
- model osno premikjoče se plošče,
- ...





Linearni modeli prečnega nihanja vej jermena

- Gibalna enačba osno premikajočega se viskozno dušenega prednapetega nosilca

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}-(T-mv^2)\frac{\partial^2 w(x,t)}{\partial x^2}+2vm\frac{\partial^2 w(x,t)}{\partial x\partial t}+m\frac{\partial^2 w(x,t)}{\partial t^2}+d\left(\frac{\partial w(x,t)}{\partial t}+v\frac{\partial w(x,t)}{\partial x}\right)=f(x,t)$$

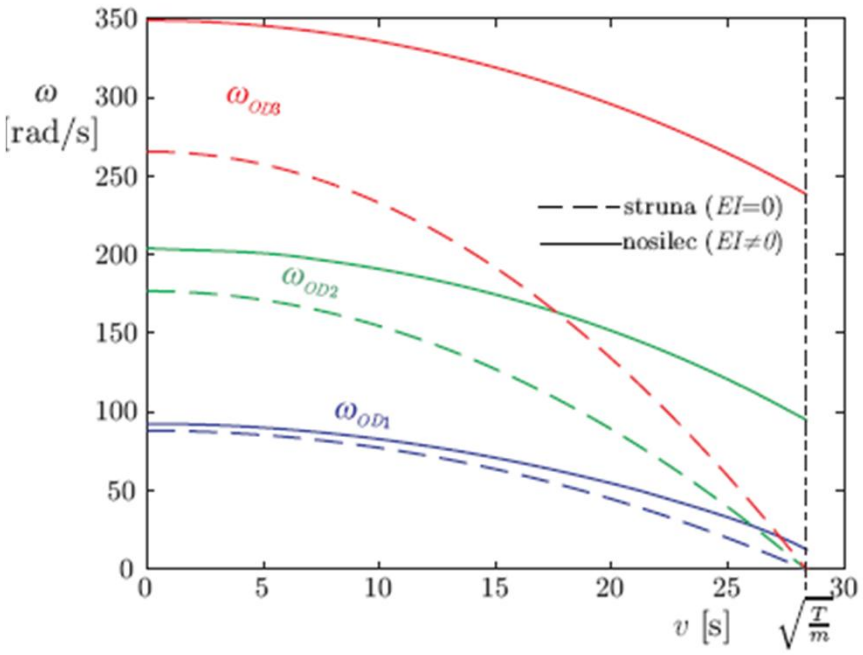
Nosilec	Gibalna enačba
$T=0$	: Osno premikajoč se viskozno dušen nosilec
$v=0$	: Viskozno dušen prednapet nosilec
$T=0,\ v=0,\ d=0$	: Nosilec
Struna	
$EI=0$	: Osno premikajoča se viskozno dušena struna
$EI=0,\ v=0$	: Viskozno dušena struna
$EI=0,\ v=0,\ d=0$	: Struna

- Lastna frekvenca osno premikajoče se viskozno dušene strune

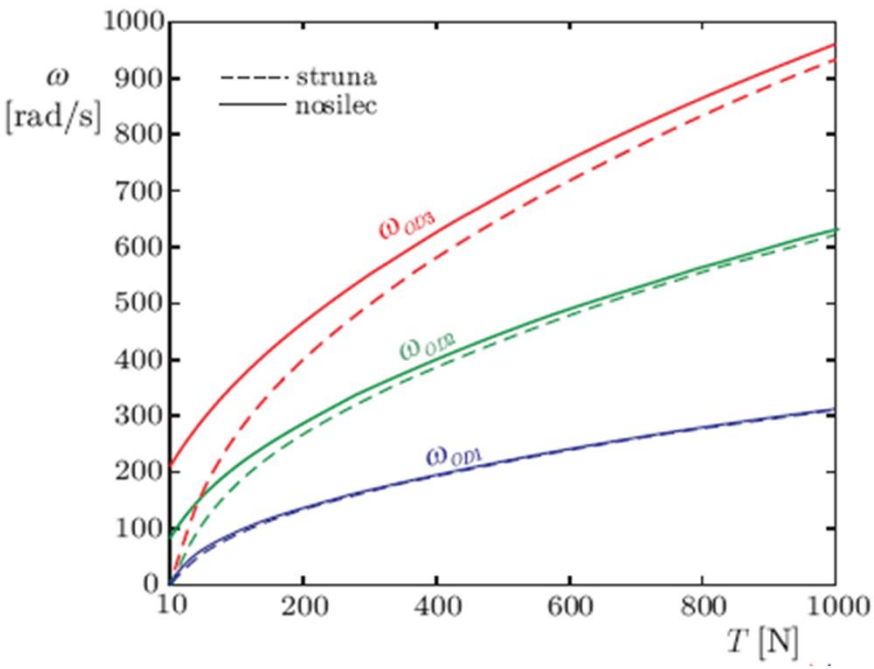
$$\omega_{0D}=\sqrt{\left(\frac{T-mv^2}{\sqrt{Tm}}\right)^2\left(\frac{n\pi}{L}\right)^2-\frac{T-mv^2}{T}\left(\frac{d}{2m}\right)^2}$$

Lastne frekvence nosilca - strune

Odvisnost lastne frekvence od osne sile

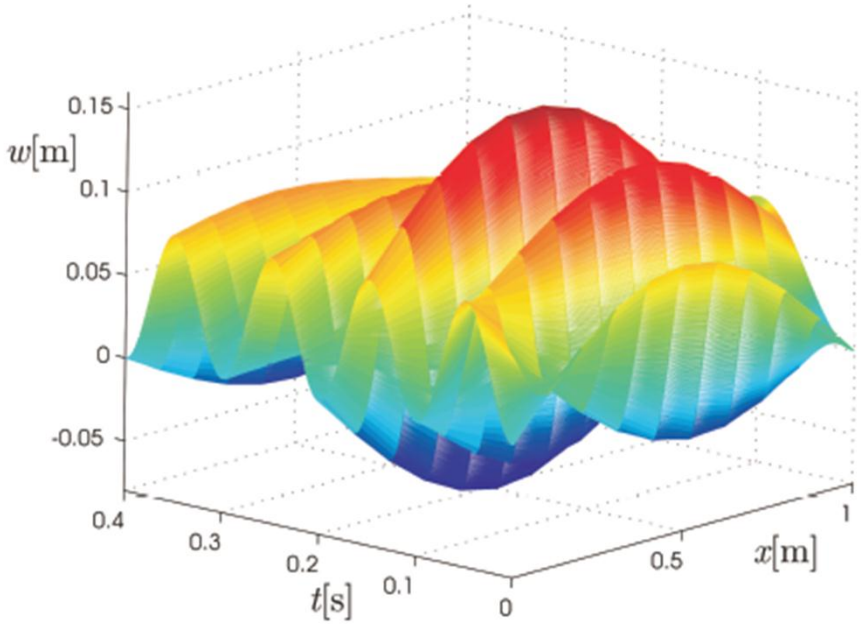


Odvisnost lastne frekvence od sile prednapetja

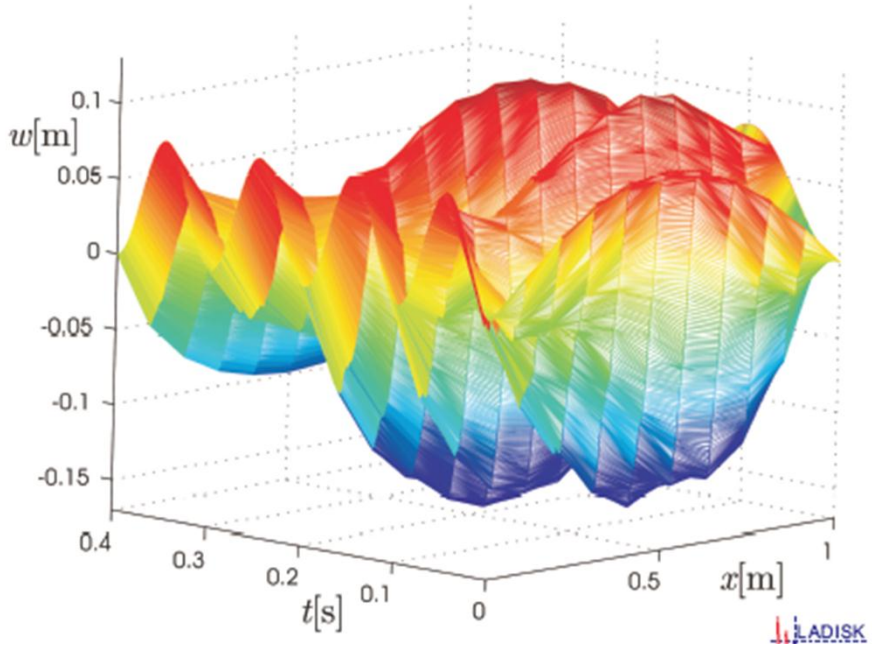


Časovni odziv nosilca - strune

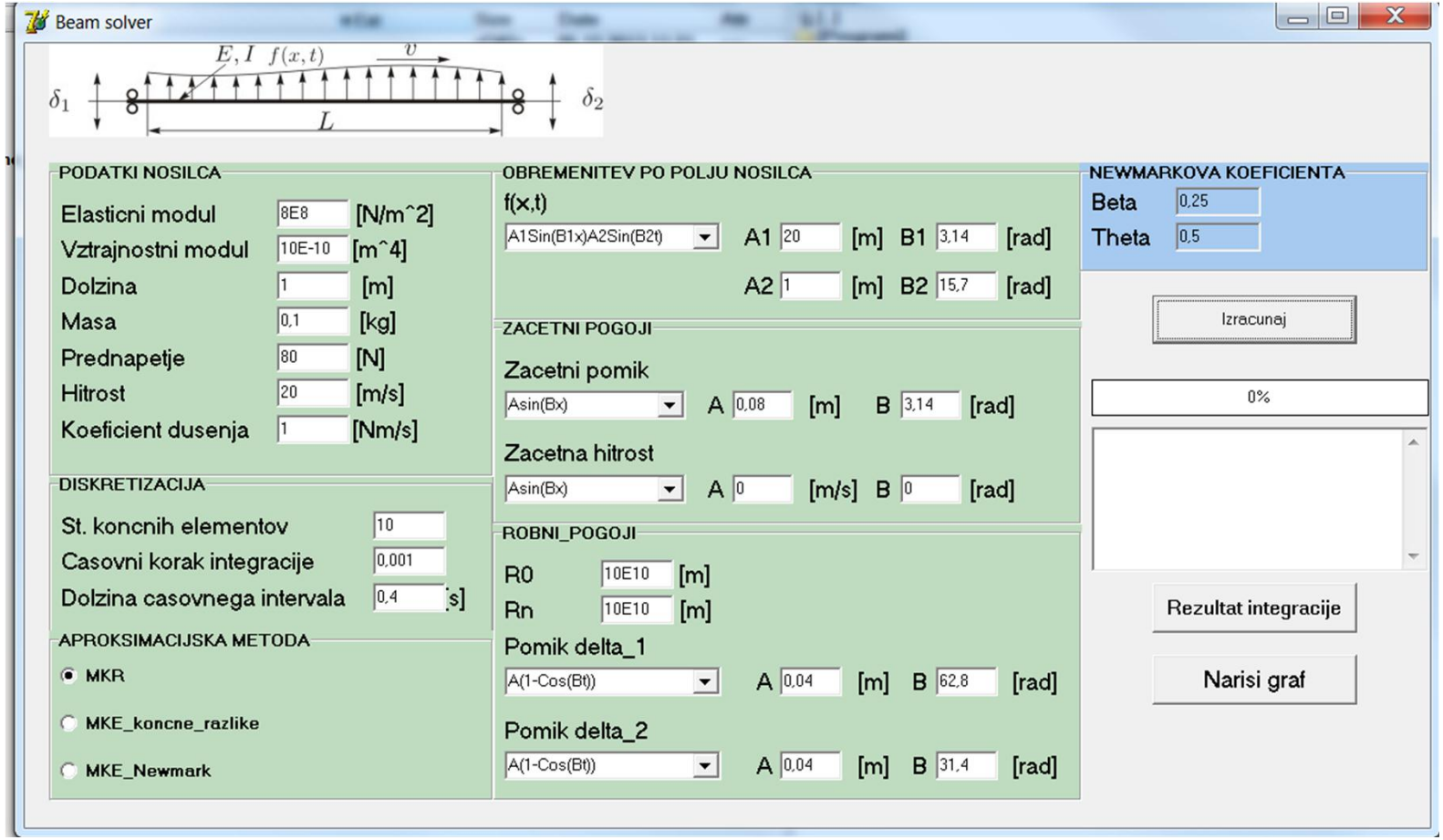
$$R_0 = \infty \Longrightarrow \partial^2 w(0,t)/\partial x^2 = 0$$
$$R_L = \infty \Longrightarrow \partial^2 w(0,t)/\partial x^2 = 0$$



$$R_0 = \infty \Longrightarrow \partial^2 w(0,t)/\partial x^2 = 1/R_0$$
$$R_L = \infty \Longrightarrow \partial^2 w(0,t)/\partial x^2 = 1/R_L$$



Časovni odziv nosilca - strune



Periodično variiranje sile prednapetja – struna (1)

- Harmonsko variiranje sile prednapetja $T = T_0 + \Delta T cos(\omega_e t)$
- Gibalna enačba osno premikajoče se strune

$$\left(p_1^2 + \Delta p_1^2 \cos(\omega_e t) - \kappa v^2\right) \frac{\partial^2 w(x,t)}{\partial x^2} - v^2 \frac{\partial^2 w(x,t)}{\partial x^2} - 2v \frac{\partial w(x,t)}{\partial x \partial t} - \frac{\partial^2 w(x,t)}{\partial t^2} = 0,$$

$$p_1 = \sqrt{\frac{T_0}{m}}, \quad \Delta p_1 = \sqrt{\frac{\Delta T}{m}}, \quad \kappa = 1 - \eta, \quad 0 \leq \eta \leq 1$$

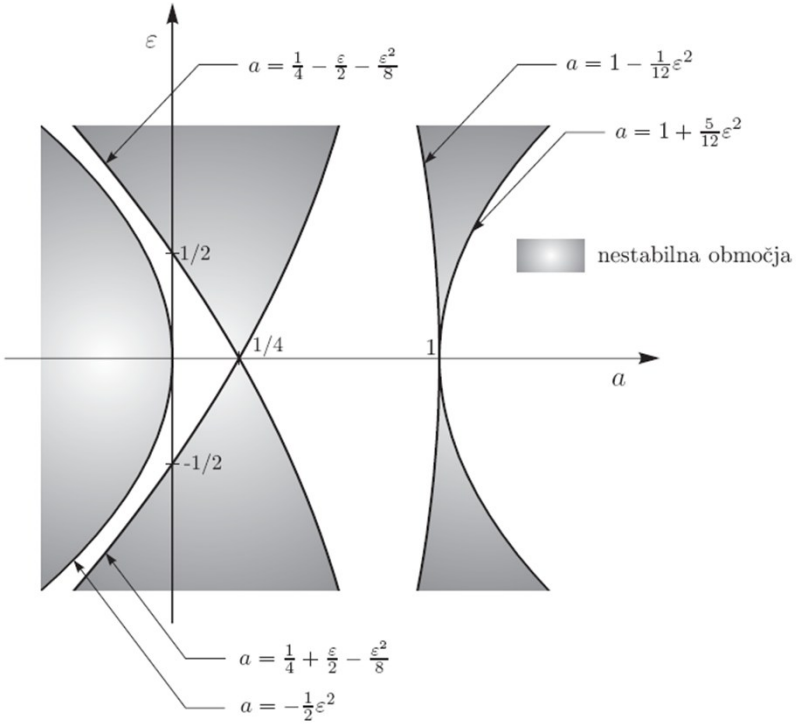
- Uporabimo nastavek $\omega(x,t) = u(t) e^{-i\alpha(x - vt)}$
- Gibalno enačbo prevedemo na Mathieujevo enačbo

$$\frac{\mathrm{d}^2 u_i(z)}{\mathrm{d}z^2} + \left(a_i + \varepsilon_i \cos(z)\right) u_i(z) = 0, \qquad i = 1, 2$$

$$a_i = \left(\frac{n_i}{\omega_e L}\right)^2 \left(p_1^2 + \eta v^2\right), \quad \varepsilon_i = \left(\frac{n_i}{\omega_e L}\right)^2 \Delta p_1, \quad i = 1, 2$$

Periodično variiranje sile prednapetja – struna (2)

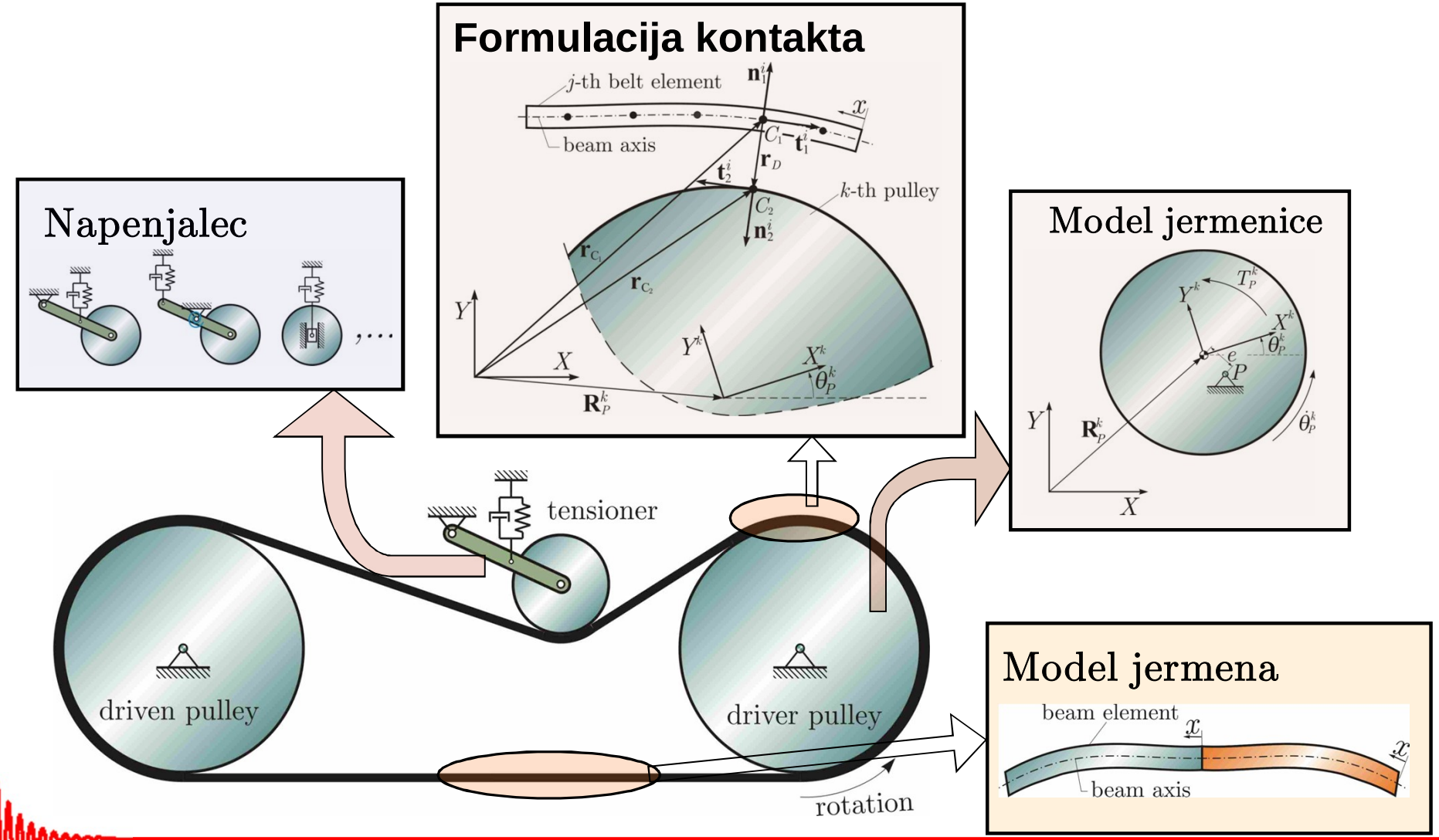
- Nestabilna območja Mathieujeve enačbe



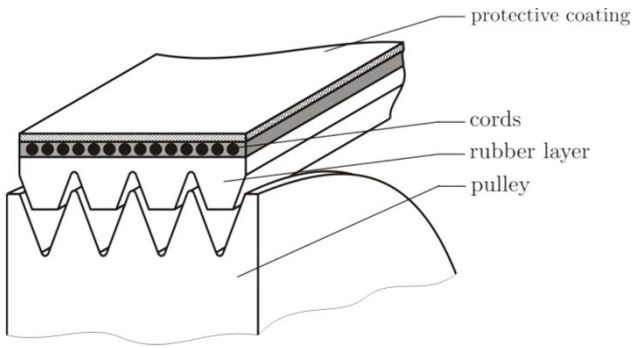
- Najverjetnejša točka nestabilnosti

$$a_1 = \frac{1}{4}: \quad \left(\frac{\omega}{\omega_e}\right)^2 = \frac{1}{4}\left(\frac{p_1^2 - \kappa v^2}{v\sqrt{p_1^2 + \eta v^2} \pm (p_1^2 + \eta v^2)}\right)^2$$

Modeliranje v okviru dinamike togih in fleksibilnih teles



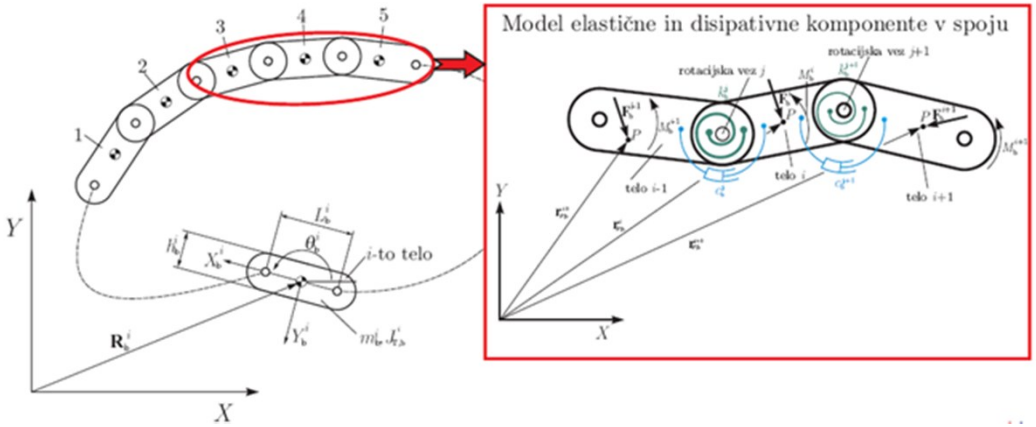
Model jermena



- Viskoelastični material jermena

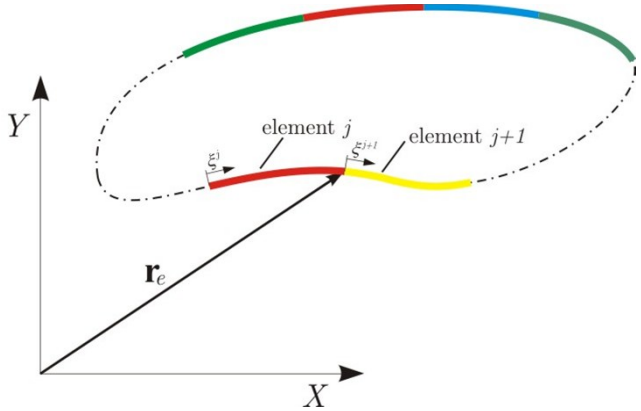
$$\sigma_1(x,t) = E\,\varepsilon_1(x,t) + c\,\frac{\partial \varepsilon_1(x,t)}{\partial t}$$

- Različni modeli jermena (toga deformabilna telesa)



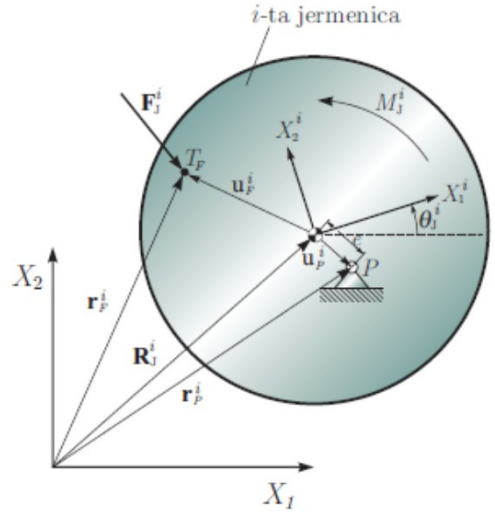
- Gibalna enačba

$$\begin{bmatrix} \mathbf{M}_{\mathrm{B}} & \mathbf{C}_{\mathbf{e},\mathrm{B}}^{\mathrm{T}} \\ \mathbf{C}_{\mathbf{e},\mathrm{B}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{e}} \\ \boldsymbol{\lambda}_{\mathrm{B}} \end{Bmatrix} = \begin{bmatrix} \mathbf{Q}_{\mathrm{f}} + \mathbf{Q}_{\mathbf{e},\mathrm{B}} \\ \mathbf{Q}_{\mathrm{d},\mathrm{B}} \end{bmatrix}$$

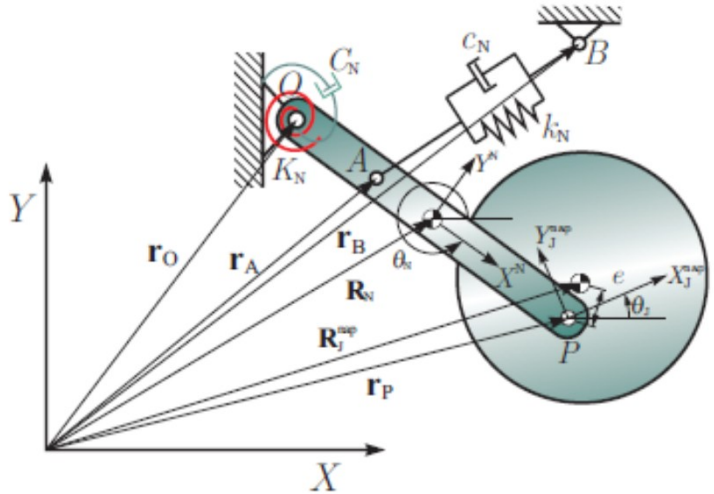


Model jermenic – napenjalnik jermenic

- Model jermenice



- Model napenjalne jermenice



$$\mathbf{C}_{\mathbf{q},JN} = \left[\begin{array}{c|c} \mathbf{C}_{q,J} & \begin{matrix} \vdots \\ -\left[\mathbf{I}, \mathbf{A}_\theta^N \mathbf{u}_P^N\right] \\ \vdots \end{matrix} \\ \hline \mathbf{0} & \mathbf{C}_{q,N_1} \end{array}\right]$$

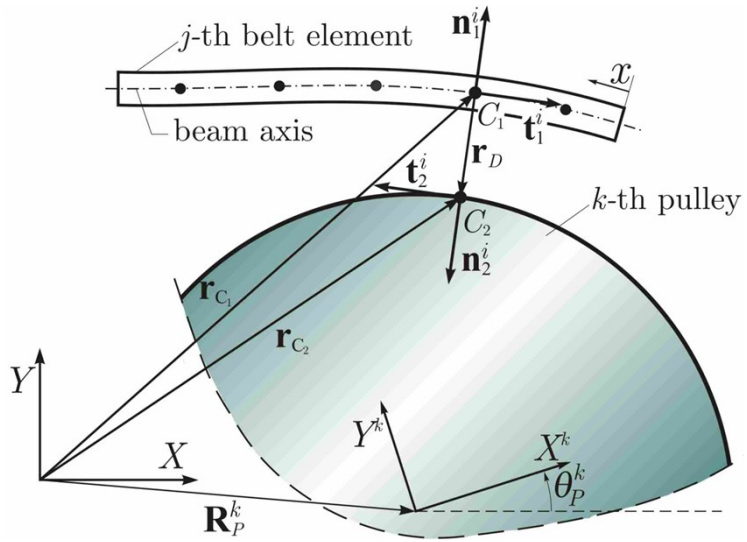
Formulacija kontaktnega problema (1)

- Kontaktna kinematika

$$\ddot{\mathbf{g}}_{\mathsf{N}} = \mathbf{W}_{\mathsf{N}}^{\mathsf{T}}\ddot{\mathbf{q}} + \mathbf{w}_{\mathsf{N}}^{\mathsf{T}},$$
$$\ddot{\mathbf{g}}_{\mathsf{T}} = \mathbf{W}_{\mathsf{T}}^{\mathsf{T}}\ddot{\mathbf{q}} + \mathbf{w}_{\mathsf{T}}^{\mathsf{T}},$$

- Gibalna enačba s kontaktnimi silami

$$\ddot{\mathbf{q}}_{\mathsf{r}} = \mathbf{H}_{\mathsf{F}}\left[\mathbf{W}_{\mathsf{N}}, \mathbf{W}_{\mathsf{T}}\right]\left\{\begin{array}{c}\boldsymbol{\lambda}_{\mathsf{N}}\\ \boldsymbol{\lambda}_{\mathsf{T}}\end{array}\right\} + \mathbf{h}.$$



Normalne kontaktne sile

Trčni kontakti ($\ddot{g}_{\mathsf{N}} \neq 0$)

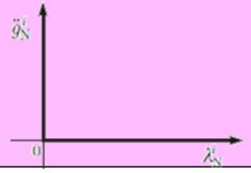
Penalna metoda:

$$\lambda_{\mathsf{N}}^{i} = F_{\mathsf{el}}(g_{\mathsf{N}}^{i}) + \mathsf{H}(\dot{g}_{\mathsf{N}}^{i})F_{\mathsf{dis}}(\dot{g}_{\mathsf{N}}^{i})$$
$$\mathsf{H}(x) = \begin{cases} 1, & x \leq 0 \\ 0, & x > 0 \end{cases}.$$

Trajni kontakti ($\ddot{g}_{\mathsf{N}} = 0$)

Kontaktni zakon:

$$\ddot{\mathbf{g}}_{\mathsf{N}} \geq \mathbf{0}, \quad \boldsymbol{\lambda}_{\mathsf{N}} \geq \mathbf{0}, \quad \ddot{\mathbf{g}}_{\mathsf{N}}^{\mathsf{T}}\boldsymbol{\lambda}_{\mathsf{N}} = \mathbf{0}$$



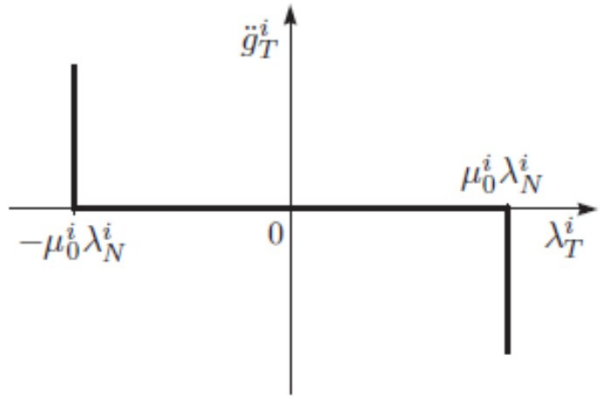
Formulacija kontaktnega problema (2)

- Modeliranje trenja

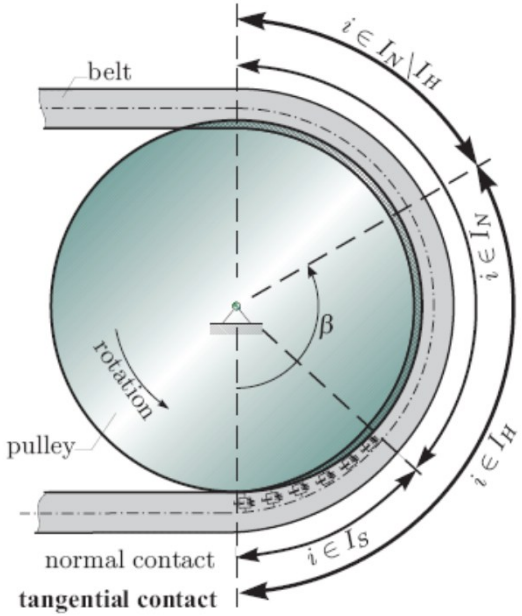
$$\left|\lambda_{\mathrm{T}}^{i}\right| \leq \mu_{0}^{i}\lambda_{\mathrm{N}}^{i} \quad \Rightarrow \quad \ddot{g}_{\mathrm{T}}^{i} = 0$$ lepenje

$$\lambda_{\mathrm{T}}^{i} = \mu_{0}^{i}\lambda_{\mathrm{N}}^{i} \quad \Rightarrow \quad \ddot{g}_{\mathrm{T}}^{i} < 0$$ negativno drsenje

$$\lambda_{\mathrm{T}}^{i} = -\mu_{0}^{i}\lambda_{\mathrm{N}}^{i} \quad \Rightarrow \quad \ddot{g}_{\mathrm{T}}^{i} > 0$$ pozitivno drsenje

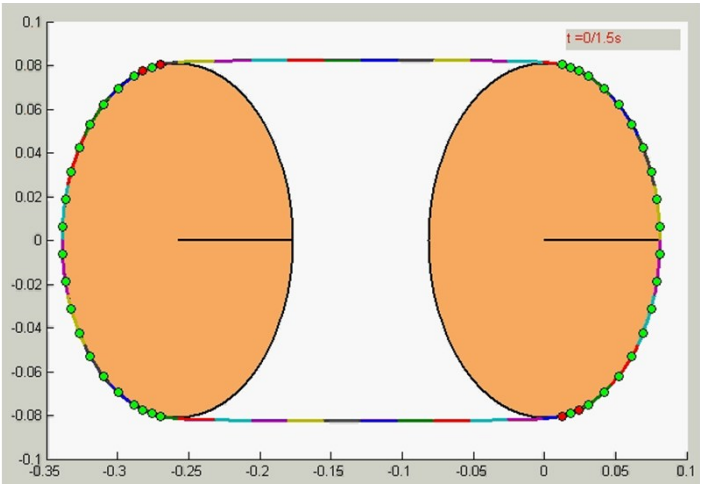


- Identifikacija območij lepenja in drsenja

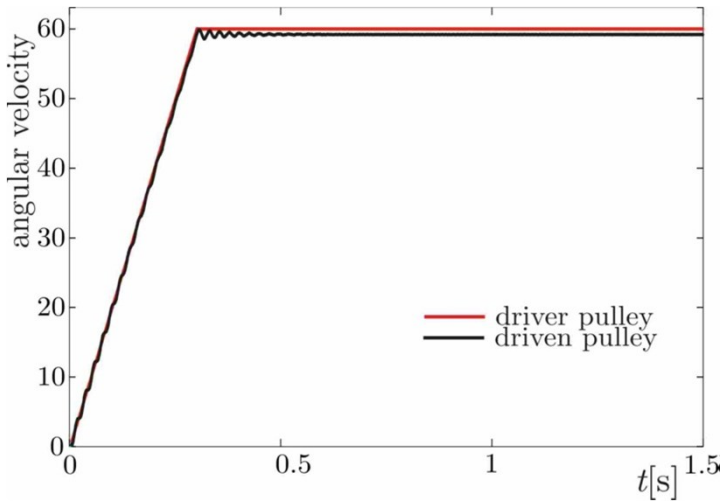


Simuliranje drsnih območij – enakomerno obratovanje

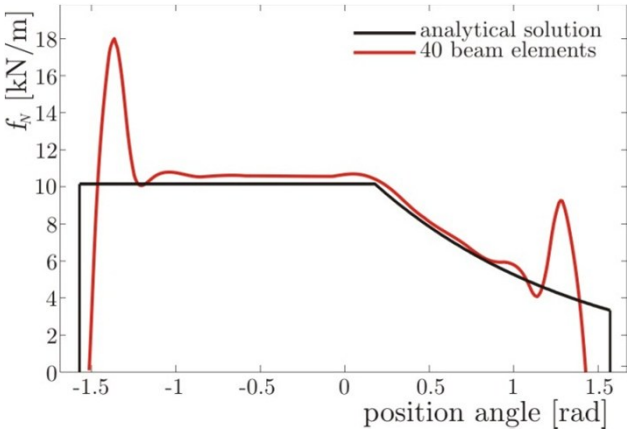
Dinamični odziv



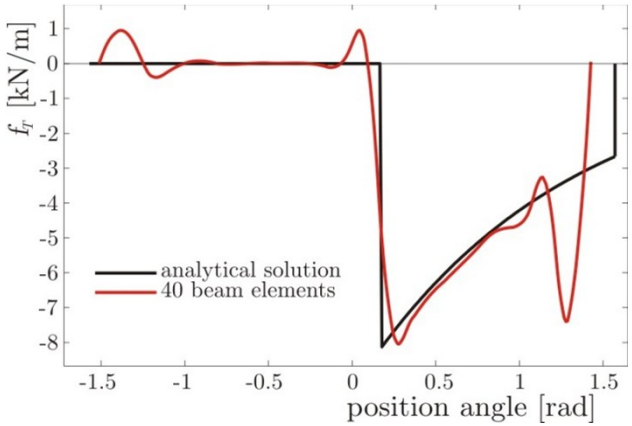
Kotna hitrost



Normalne kontaktne sile

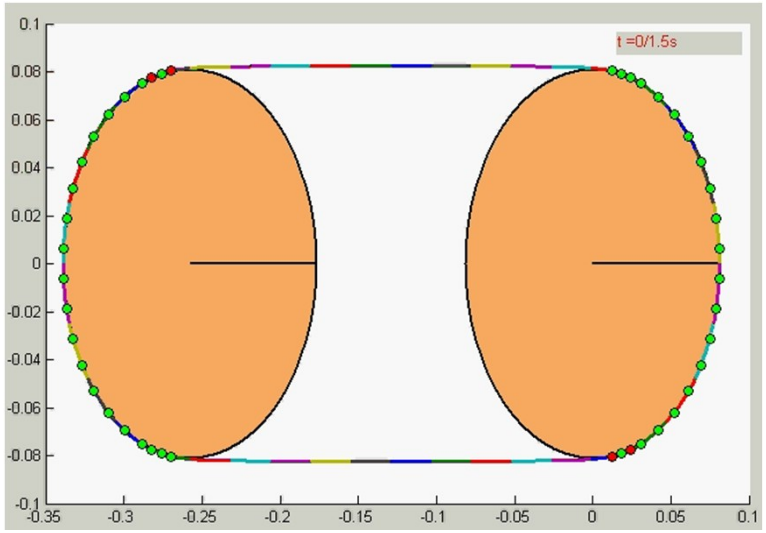


Sila trenja

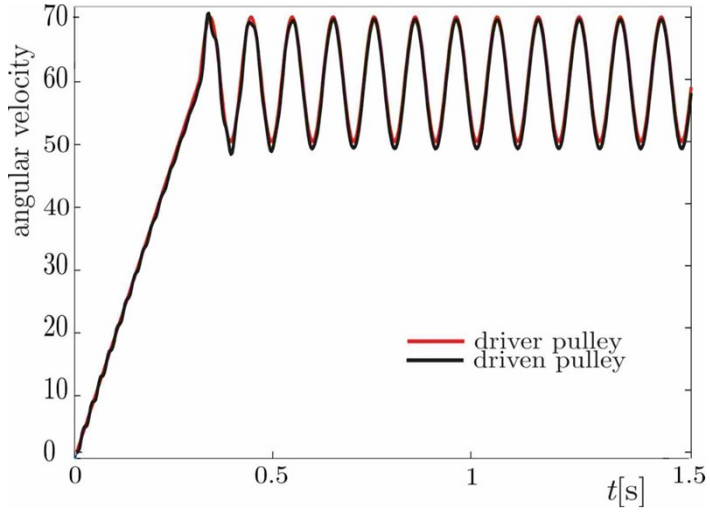


Simuliranje drsnih območij – neenakomerno obratovanje

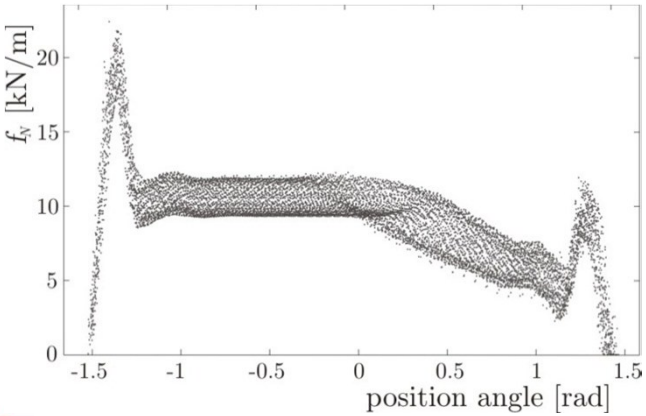
Dinamski odziv



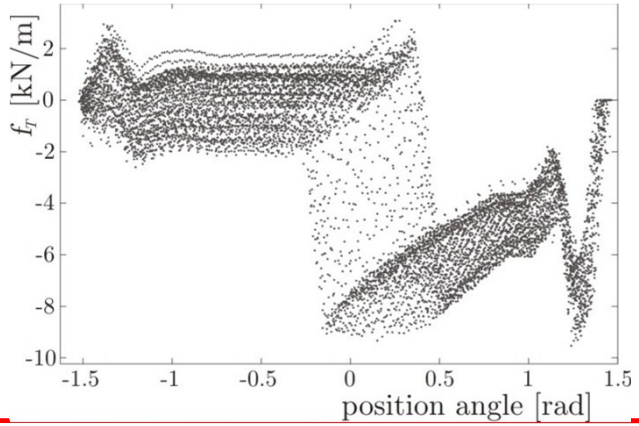
Kotna hitrost



Normalne kontaktne sile

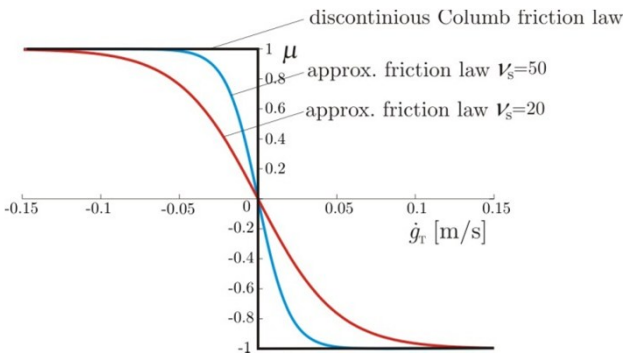


Sila trenja

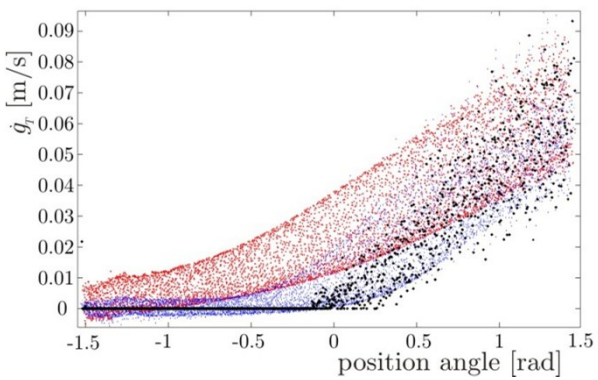


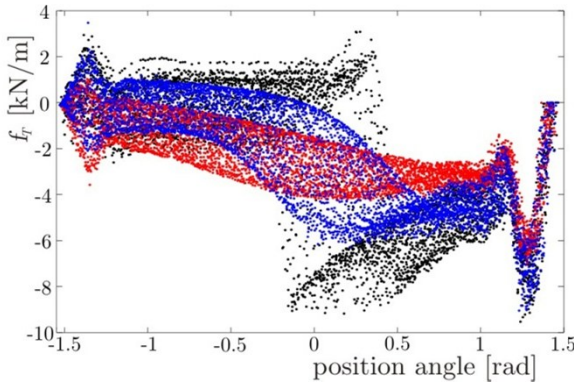
Velikost drsnega območja

- Zakon trenja



- Relativna tangentna hitrost





- Računski časi

	discontinious Coulomb friction law	approximate friction law		
		$\nu_s = 10$	$\nu_s = 20$	$\nu_s = 50$
CPU time [h]	20	30,4	77,2	150

Identifikacija materialnih lastnosti

- V-rebrasti jermen (5pk1200) s petimi rebri

- Materialne lastnosti
  - Togost in dušenje v osni smeri
  - Togost in dušenje v prečni smeri
- Kontaktni parametri
  - Kontaktna togost
  - Koeficient trenje

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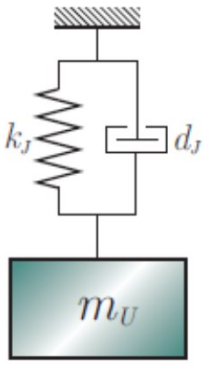
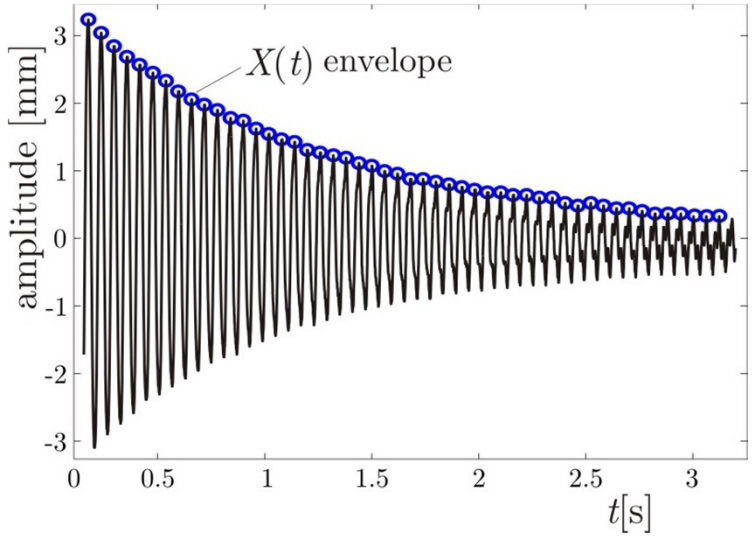
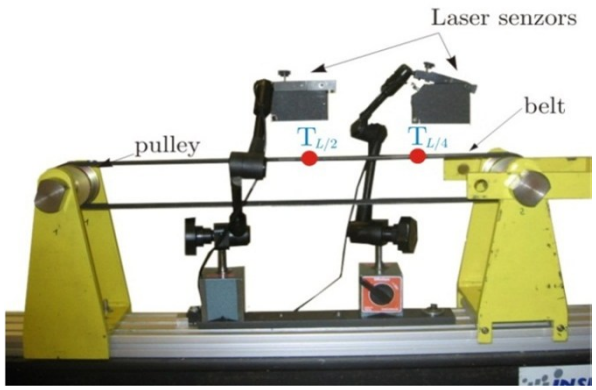
Identifikacija osne in upogibne togosti in dušenja

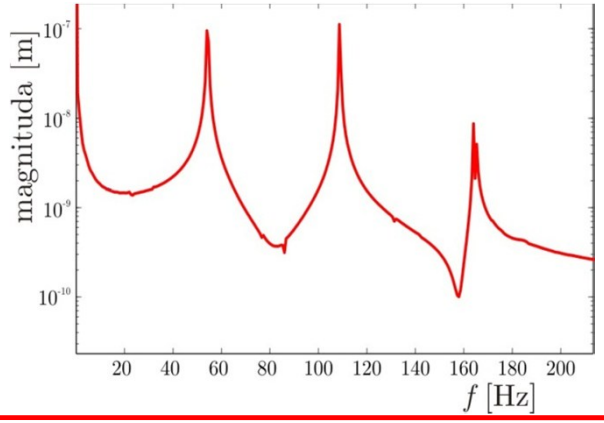
- Identifikacija osne togosti in dušenja na osnovi lastnega odziva jermena





- Identifikacija upogibne togosti in dušenja na osnovi lastnega odziva jermena

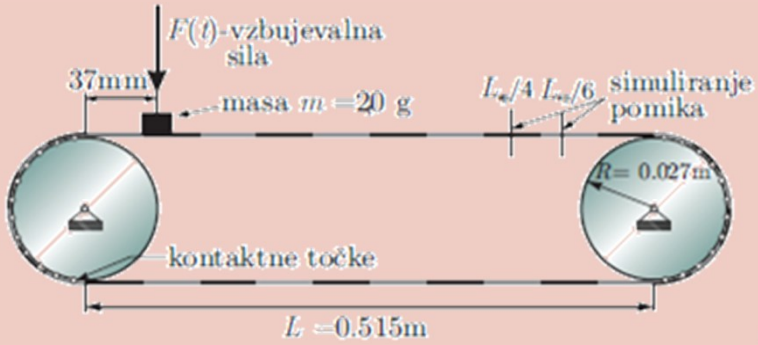




Odziv jermena v prečni smeri

Identifikacija dušenja v prečni smeri

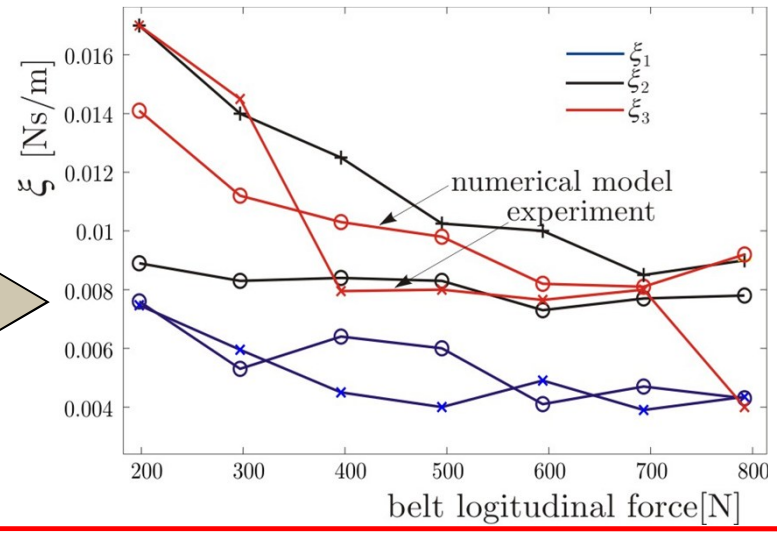
Numerični eksperiment



Eksperimentalno identificirani $\xi^{\exp}$

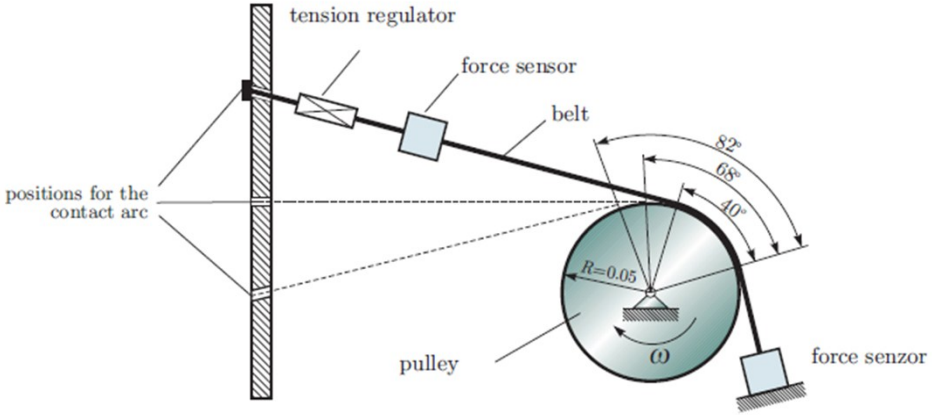
Optimizacija

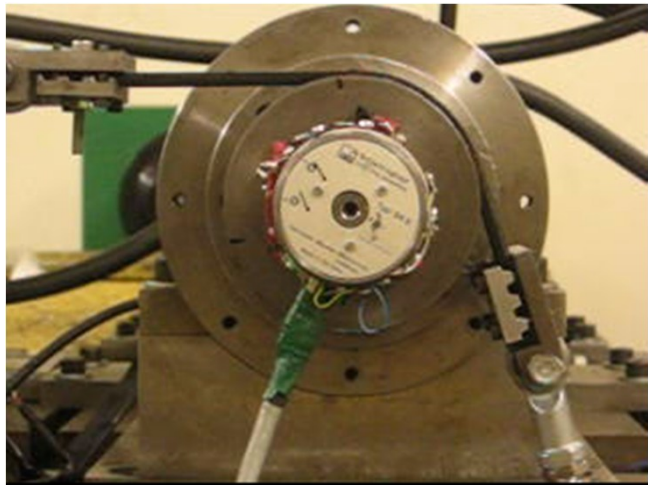
$$\min_{\alpha,\beta} \frac{1}{2} \sum_k \left| \begin{pmatrix} \xi_1(\alpha,\beta,T_k) - \xi_1^{\text{eksp}}(T_k) \\ \xi_2(\alpha,\beta,T_k) - \xi_2^{\text{eksp}}(T_k) \\ \xi_3(\alpha,\beta,T_k) - \xi_3^{\text{eksp}}(T_k) \end{pmatrix} \right|^2$$



Merjenje sile trenja

- Eksperiment



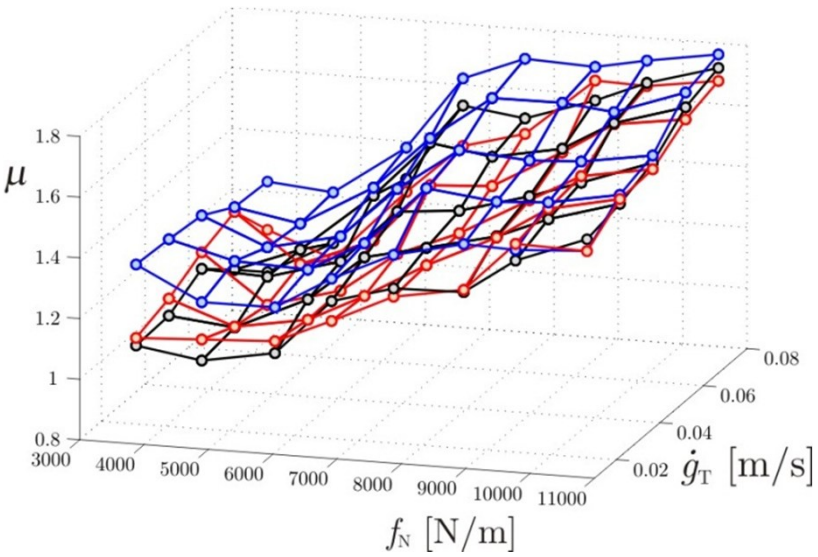


- Koeficient trenja

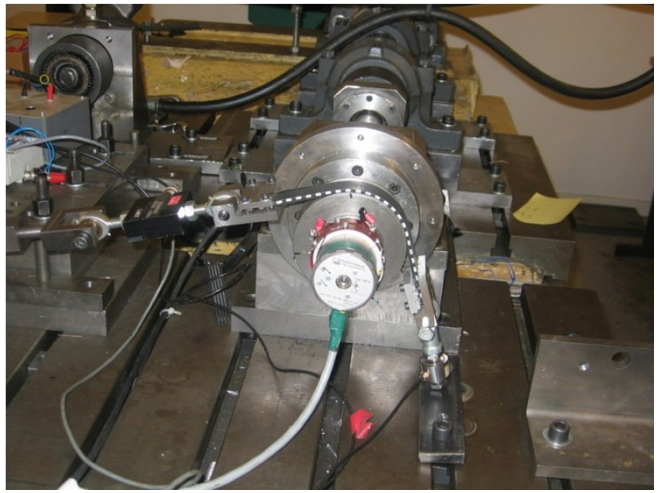
$$\mu = \frac{1}{\beta_1} \ln\left(\frac{T_\mathrm{d}}{T_\mathrm{j}}\right)$$

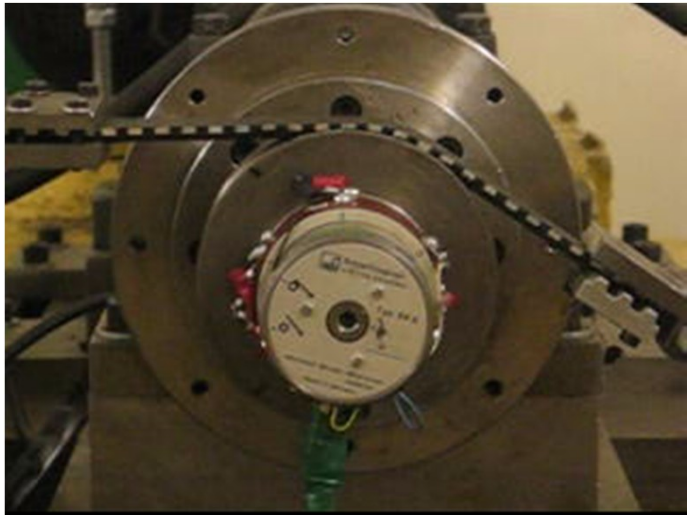
- Normalna kontaktna sila

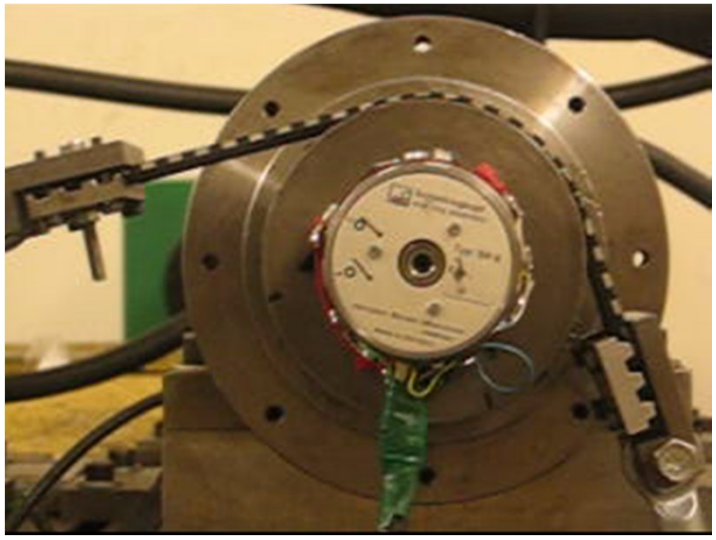
$$f_\mathrm{N} = \frac{1}{R} T_\mathrm{d} \mathrm{e}^{-\mu_\mathrm{ref}\phi}, \qquad 0 \le \phi \le \beta$$



Merjenje sile trenja - eksperiment





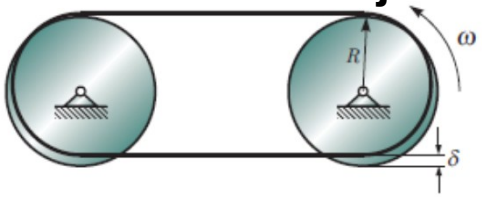


Zmanjšanje kotne hitrosti gnane jermenice

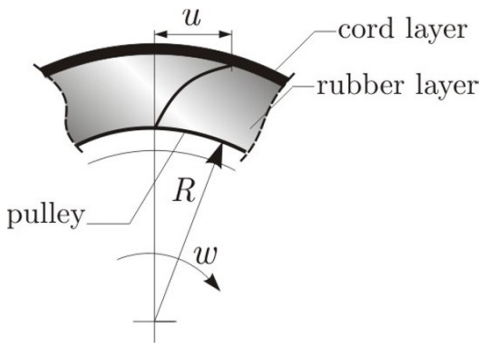
Zdrs jermena
(G. Gerbert, Belt Slip-A Unified Approach)

Lezenje

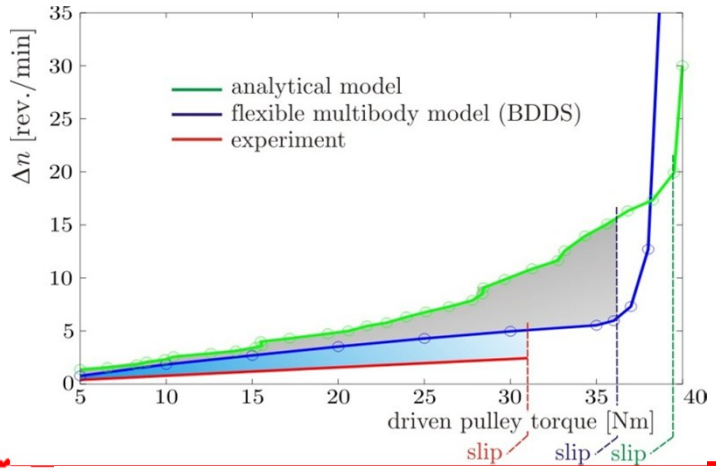
Radialna deformacija



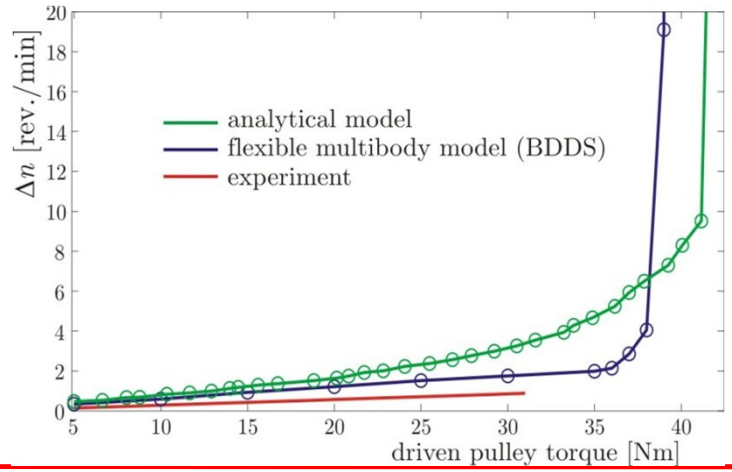
Tangentna deformacija



$\omega = 62{,}8\,\mathrm{rad/s}$, $T_0 = 310\,\mathrm{N}$

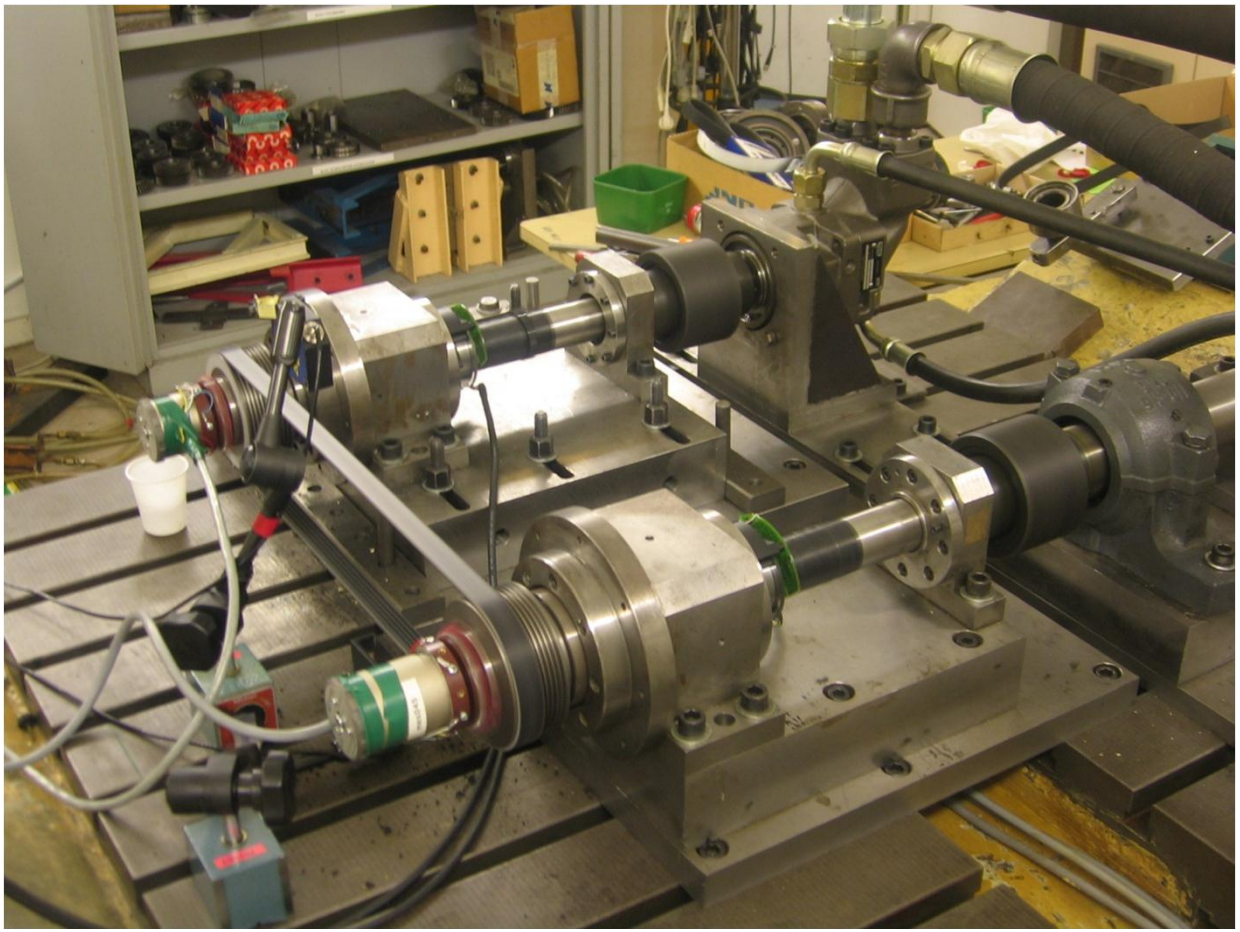


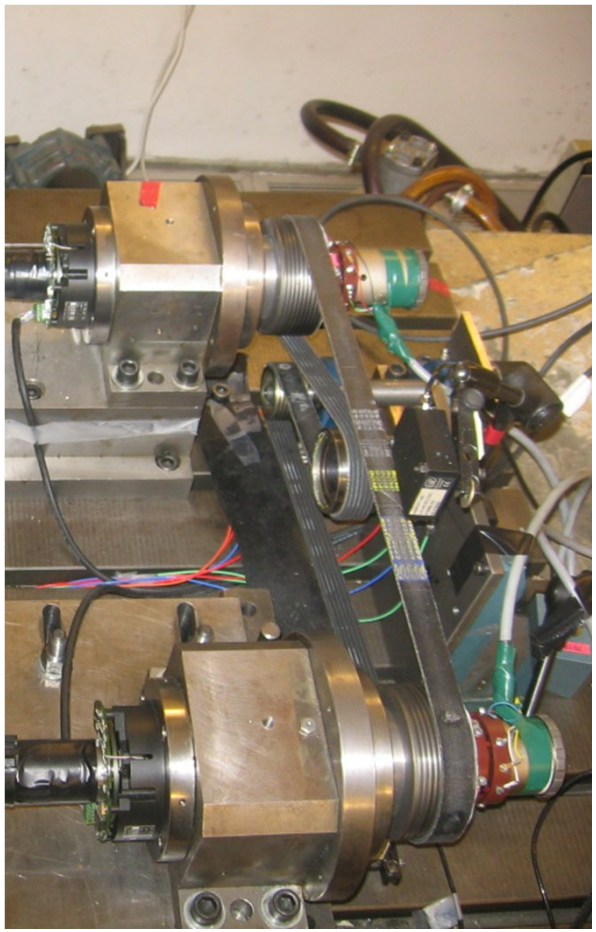
$\omega = 22{,}6\,\mathrm{rad/s}$, $T_0 = 310\,\mathrm{N}$



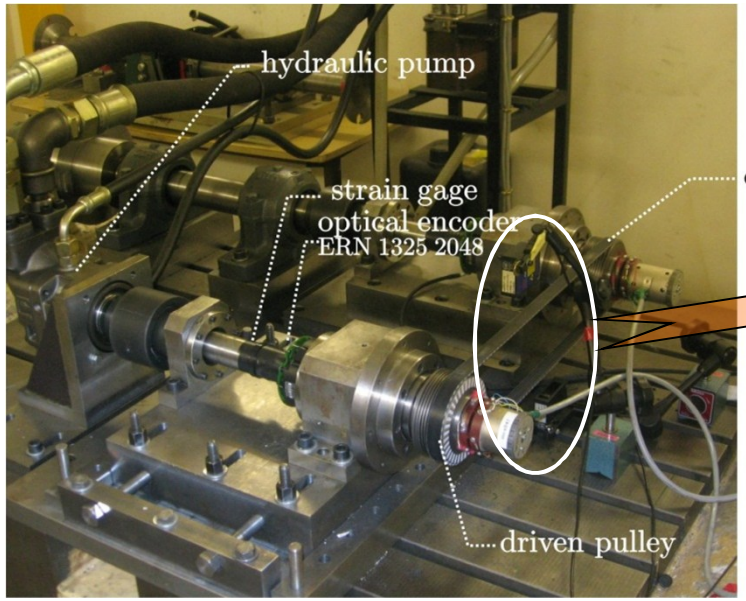
Eksperimentalna identifikacija zdrsa jermena (1)

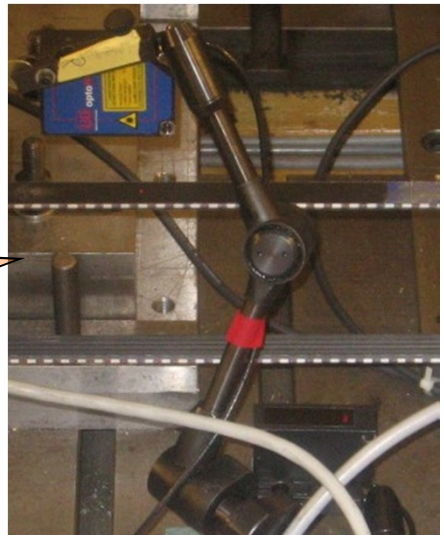
Konstrukcija preizkuševališča

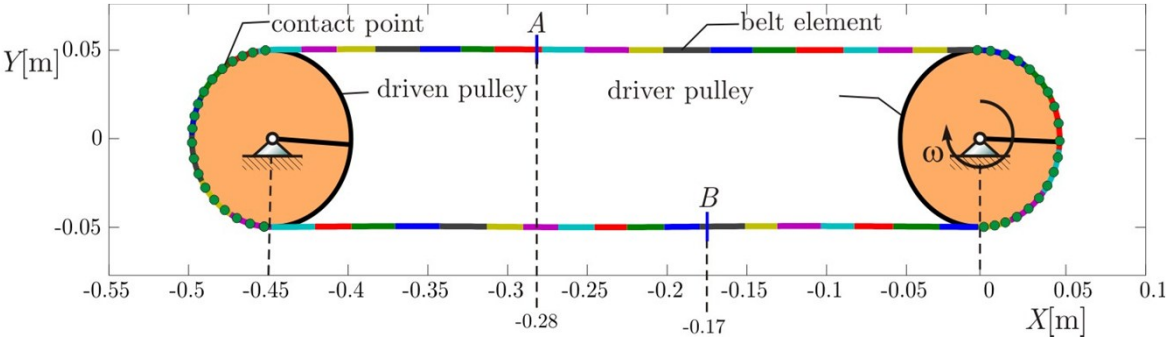




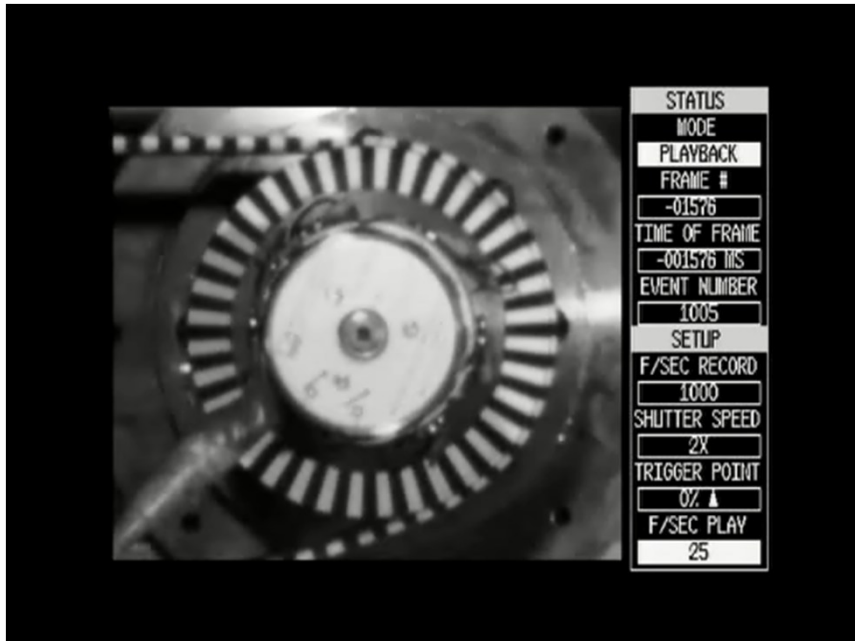
Eksperimentalna identifikacija zdrsa jermena (2)

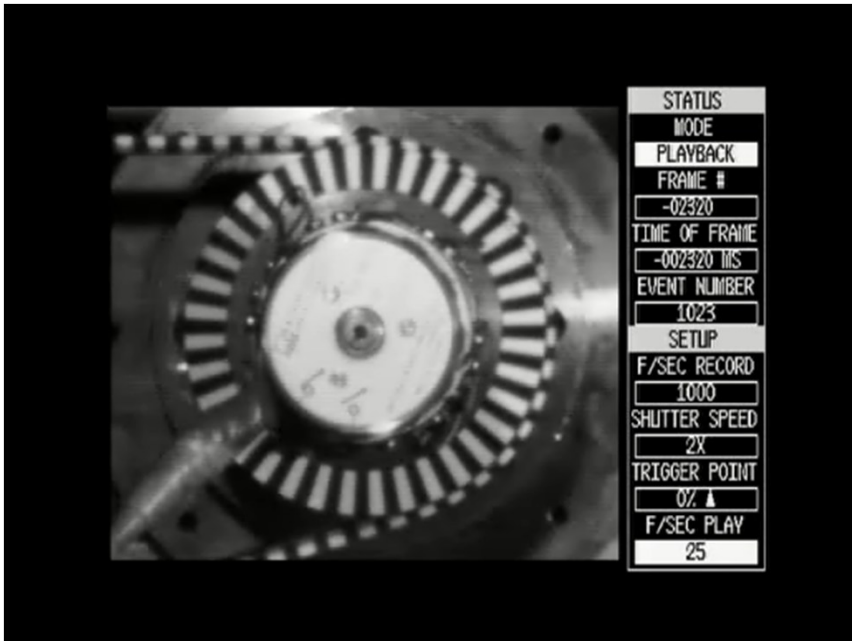




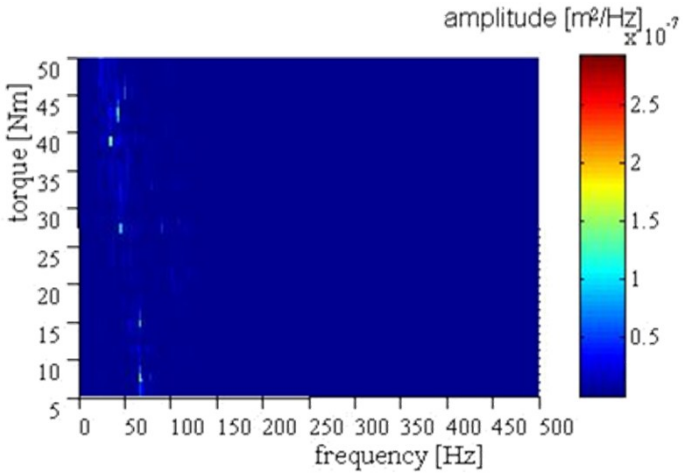
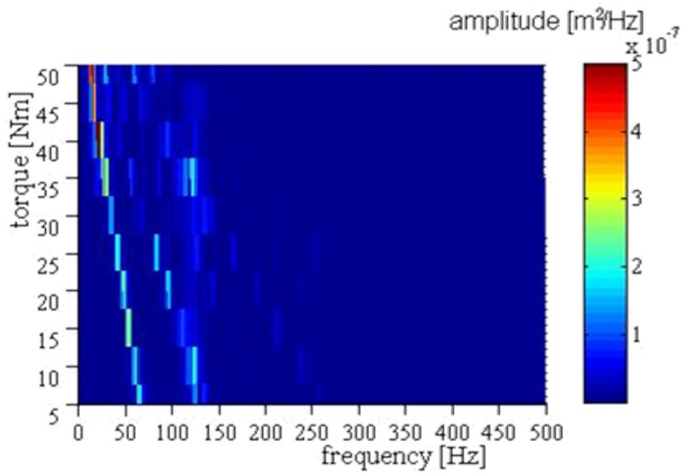
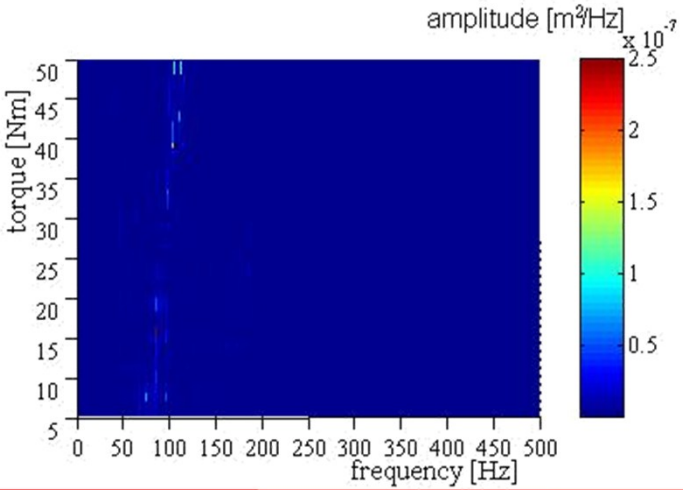
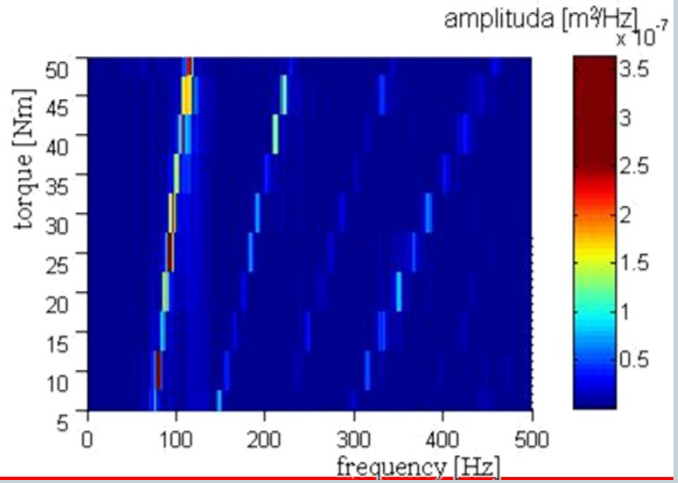


Eksperimentalna identifikacija zdrsa jermena (3)

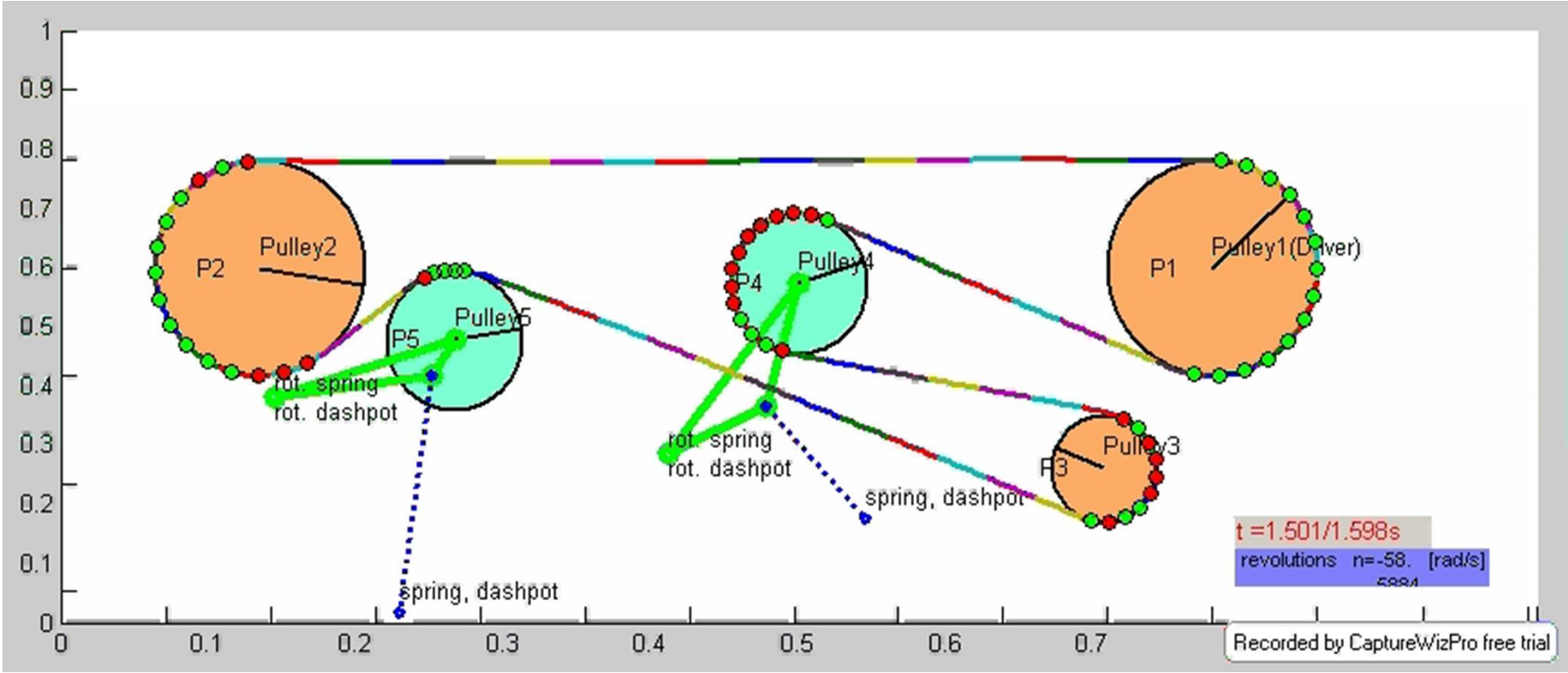




Prečna nihanja veje jermena

	Eksperiment	Numerični model
Slack span		
Tight span		

Kompleksna jermenska gonila



Hvala za pozornost.

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Laboratorij za dinamiko strojev in konstrukcij

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