Vibration-Fatigue Damage Accumulation for Structural Dynamics with Non-linearities

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Abstract

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Structural damage in mechanical components is frequently caused by high-cycle vibration fatigue. The non-linearities, frequently observed in real structures at increased excitation levels, significantly influence the damage accumulation. As the modal analysis bases on linear theory, the non-linearities are hard to include. Based on a new experimental identification of the non-linearities, this research proposes the corrected linear damage-accumulation estimation. With the proposed correction, the linear modal analysis is used for damage estimation of structures with non-linearities.

The proposed approach is applied to a real-life case of steel-sheet attached with rivets. Several samples are exposed to an accelerated vibration-fatigue test with increasing and also decreasing excitation levels. It is shown that with the experimentally identified non-linearity correction, the numerical fatigue life-time was within the 10% of the experimentally identified life-time. Experimentally, it was shown that rivets same by design, but produced by different manufacturers, have a significant difference in the fatigue life-time; this difference was clearly identified with the proposed correction to the linear damage-accumulation estimation.

Further, the frequency response function based identification of the non-linearity can be identified before the structure is exposed to fatigue loads resulting in new possibilities of vibration-fatigue analysis of non-linear systems.

1 Introduction

Fatigue failures in metallic structures are a well-known technical problem [1]. Fatigue damage increases with the applied load cycles in a cumulative manner. Cumulative fatigue-damage analysis plays a key role in the life-time prediction of components and structures subjected to field load histories [2]. Probably the first systematic fatigue testing was undertaken by Wöhler [3], who concluded that the cyclic stress range is more important than the peak stress and introduced the concept of an endurance limit. Palmgren [4] introduced the first damage-accumulation theory, which is now known as the linear rule, and then Miner [5] expressed the linear rule in mathematical form as a summation of the damage with different loadings, which were calculated as the ratio between the number of applied cycles and the number of total cycles until failure for the ith constant-amplitude loading level. Recently, based on a linear accumulation rule, Zambrano and Foti [6] proposed damage indices for predicting the life of aerospace structures or seismic-resistant structures subjected to a low-cycle fatigue phenomenon. Marco and Starkey [7] proposed a non-linear, load-dependent damage rule based on an exponentiated Miner's damage accumulation of the coefficients depending on the ith load level. A different approach, based on the damage mechanics of continuous media, was extensively researched [8, 9, 10, 11, 12, 13]. This approach deals with the mechanical behaviour of the deteriorating medium on a continuum scale with regards to the original concept of Rabotnov [8] and Kachanov [9]. Chaboche and Lemaitre [10, 11] applied these principles to formulate a non-linear damage accumulation by including the effects of the load level. Golub [12] introduced the non-linear uniaxial damage theory and compared it with the existing models for some structural materials under long-term static and cyclic loading. Dattoma et al. [13] proposed and experimentally validated a new formulation for damage propagation with coefficients that include material and load parameters. Aid et al.[14] proposed a new damage model for a fatigue-life prediction under variable loading based on the stress history, which includes a load-dependent non-linear damage accumulation. A method of fatigue-damage accumulation based upon the application of energy parameters for the fatigue process is proposed by Djebli et al. [15]. It requires only a knowledge of the W–N curve (W: strain energy density N: number of cycles at failure) determined from the experimental Woehler curve. Huffman and Beckman [16] used the strain-load history in their approach to predict the fatigue life-time due to the damage-accumulation process at different loading levels.

The majority of the above research is focused on low-cycle fatigue, while high-cycle, vibration fatigue, related to structural dynamics is the focus of this research. Fatigue life-time and damage-accumulation issues in relation to the change of the structural dynamics, i.e., the structure's frequency-response functions, were extensively studied [17, 18, 19, 20, 21]. Shang et al. [17, 18] experimentally and numerically investigated the effects of fatigue-damage-accumulation on the natural frequency response. They identified a non-linear change of the natural frequencies as the damage propagates. Shang [18] proposed a new damage-accumulation variable according to the non-linear change of the natural frequencies. Dattoma et al. [19] dealt with the application of a technique based on the natural frequencies and damping to predict the fatigue failure of closed-cell aluminium foams. Česnik et al. [20] presented an improved accelerated fatigue-testing methodology based on the dynamic response of the test specimen to the harmonic excitation in the near-resonant area with simultaneous monitoring of the modal parameters. Hu et al. [21] carried out vibration tests on stiffened aluminium plates with fully clamped boundaries under random base excitation. They calculated the damage accumulation based on the rainflow-cycle counting technique of the strain history and the Miner [5] linear damage-accumulation model. They utilized the change of the natural frequencies for fitting the non-linear damage-accumulation model of the plate. Recently, Benkabouche et al.[22] developed a numerical tool for determining a non-linear, cumulative, fatigue-damage evaluation. They took into account the effects of the amplitude and the sequence of the variable amplitude loading. Owsinski and Nieslony[23] reviewed the current state of the art for durability tests performed on electromagnetic shakers.

This research deals with the effects of the structure's non-linearity on the damage accumulation. Non-linearity is excitation-amplitude dependent and estimated via the change of the amplitude in the experimentally measured frequency-response functions (Section 2). The numerical investigation of Section 3 is performed to obtain the linear damage accumulation of the structure, which is combined with the non-linearity rates to obtain the real damage accumulation used for the life-time assessment. The accelerated vibration tests presented in Section 4 are performed to validate the numerical model using the operational modal analysis, identify the real fatigue lifetimes and the non-linearity of the structures at different excitation levels. The results and discussion are presented in Sections 4 and 5.

2 Theoretical Background

One of the most common causes of structural damage is material fatigue due to dynamic loads[24]. The damage due to the vibration-fatigue process does not occur instantly. It starts with an initial crack in the places of stress concentration and propagates to the total damage [25]. Initially, the linear damage-accumulation theory is presented, which will support the later introduced non-linearity to the damage accumulation.

Here, for the damage-accumulation analysis, the high-cycle-fatigue, stress-based method is used [3, 25, 26]. Wöhler [3] investigated fatigue and used the stress-cycles (S-N) curve to characterise the fatigue behaviour of materials. Wöhler's curve can be represented using Basquin's equation [27]:

$$\sigma = C\,N^{-\frac{1}{b}}, \tag{1}$$

where b is the fatigue exponent, C is the fatigue strength and N is the number of cycles. The linear damage accumulation is calculated using the Palmgren-Miner rule [4, 5]:

$$D = 1 = \sum_i D_i = \sum_{i=1} \frac{n_i}{N_i}\,, \tag{2}$$

where D is the total damage, D_i is the damage caused by the excitation with amplitude σ_i, N_i represents the number of cycles with amplitude σ_i required for the total damage and n_i is the real number of cycles.

The damage-accumulation correction rule, based on the structural non-linearity, is proposed as a corrected damage accumulation Eq. (2):

$$D_c = \sum_i D_i \cdot R^{n_i} = 1\,, \tag{3}$$

where R represents the constant correction factor, calculated in Section 4 and n is the rate of non-linearity (can be the experimentally identified from Eq.(5)) presented in the next subsection.

The experimental investigation, presented later in Chapter 4, is used for identifying the non-linearity of dynamical systems from the measured frequency-response functions (FRFs). The FRFs were calculated using the H_2 estimator [28]:

$$H_2 = \frac{S_{x'\,x'}(\omega)}{\mathrm{S}_{x'\,f'}(\omega)}\,, \tag{4}$$

where $S_{x'\,x'}(\omega)$ and $S_{x'\,f'}(\omega)$ represent the output auto-spectrum and the cross input-output spectrum, respectively. In order to avoid the uncertainty in the measurements, each FRF is estimated as the results of the averaging of ten FRFs. In order to estimate the rate of the non-linearity, experimentally obtained FRFs at different excitation levels need to be obtained. The rate of the non-linearity estimation was based on two assumptions. A FRF with a relatively low excitation level is assumed to be linear, while the changes in the FRFs at the relatively high excitation levels are assumed to be due to the structure's non-linearity. Figure 1 shows experimentally identified FRFs at relatively low and relatively high excitation levels which are considered linear and non-linear FRFs, respectively.

This research proposes a calculation of the rate of the non-linearity n as the root-mean-square error on a logarithmic scale:

$$n_i = \sqrt{\frac{\sum_{j=1}^{N}(\log_{10}(|\mathrm{FRF}_l|_j) - \log_{10}(|\mathrm{FRF}_i|_j))^2}{N}}\,. \tag{5}$$

FRF_l and FRF_i represent the linear FRF and non-linear FRF, respectively. The index j represents the points on the frequency axis, while N represents the number of frequency points. The logarithmic scale is used as the changes in the FRFs are logarithmic, while the identification of the non-linearity is also on a logarithmic scale.

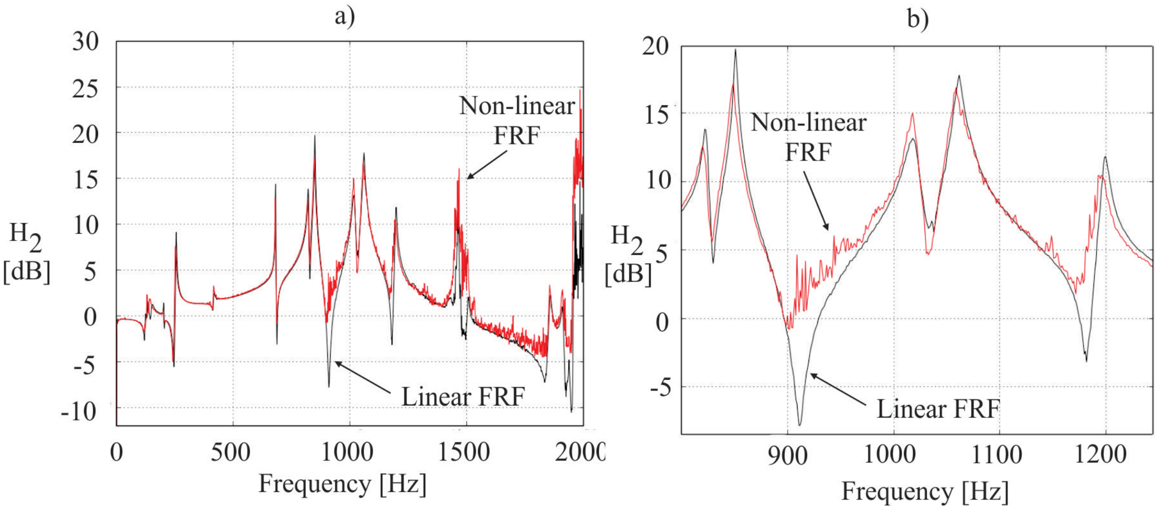

Figure 1: Linear and Non-linear FRFs; a) Whole scale b) Detail

3 The Numerical Modelling

The motivation for this investigation was a real industrial vibration-fatigue case of a thermal shield plate attached with rivets to a supporting structure. The rivets were breaking due to vibration fatigue. Figure 2 shows a simplified numerical model that corresponds to the real case and consists of the steel plate and the aluminium supporting structure, attached by three rivets. The numerical model is built from 73 518 elements and 115 700 nodes. The tetrahedral (supporting structure), hexagonal (rivets) and triangular (plate) elements are used to build the discrete dynamic model. The experimental validation of the numerical model is presented in Section 4.

With a validated numerical model the linear damage-accumulation estimation requires the following steps. The first step is to obtain the stress frequency-response function (SFRF), which is used with the excitation profile to obtain the stress response. The second step is to estimate the fatigue life-time in the frequency domain. The third step is the damage-accumulation calculation. For different excitation levels, different fatigue life-times and different damage accumulations are obtained.

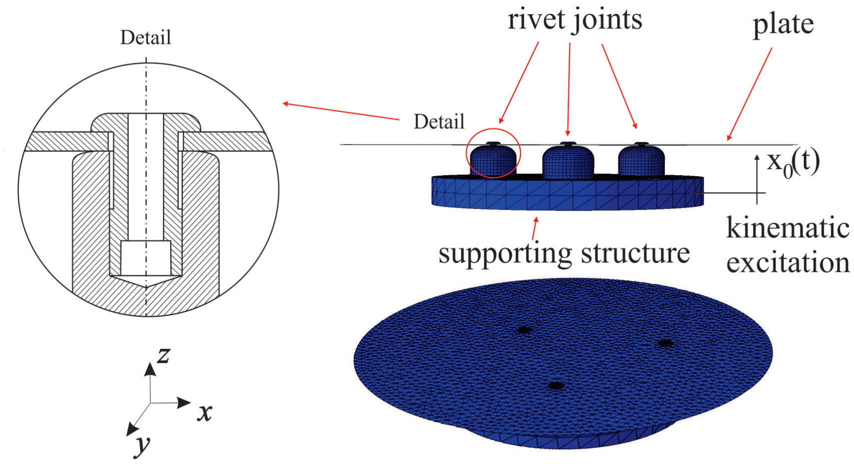

Figure 2: Dynamic - Numerical Model

The modal model, stress and displacement responses, required for the stress-response prediction of the constrained structure, are obtained from the modal analysis of the unconstrained structure (Figure 3 a)). The unconstrained structure is modified by a kinematic excitation, like in the real case, and the experimental research (Figure 3 b)), and also the model is augmented by material parameters, damping and load parameters. The extended structural modification using the response-function (SMURF) method, presented by Česnik *et. al.* [29], was used to predict the SFRF and the stress-response of the constrained structure (Figure 3 b)) using the modal model of the unconstrained structure (Figure 3 a)) in the case of a kinematic excitation.

According to [29] the equilibrium equations for a viscously damped MDOF force excited dynamic structure are:

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\} + [\mathbf{C}]\{\dot{\mathbf{x}}\} + [\mathbf{K}]\{\mathbf{x}\} = \{\mathbf{f}\}, \tag{6}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively. $\mathbf{x}$ and $\mathbf{f}$ represent the vector of total displacements and the excitation force vector, respectively. The solution to Eq. (6) is given as:

$$\{\mathbf{x}\} = [\,{}^{\mathbf{c}}\mathbf{H}(\omega)]\{\mathbf{f}\}\;, \tag{7}$$

where ω represents the frequency and ${}^{\mathbf{c}}\mathbf{H}(\omega)$ represents the constrained structure's receptance matrix, related to the experimentally estimated receptance $\mathbf{H}_2$ (Eq. 4) and calculated using the modal model of the unconstrained structure. With several transformations of Eq.(7) [29],the response of the unconstrained point is:

$$\mathbf{x_u} = ({}^{\mathbf{u}}\mathbf{H_{uc}}\,{}^{\mathbf{u}}\mathbf{H_{cc}^{-1}})\,\mathbf{x_0} + ({}^{\mathbf{u}}\mathbf{H_{uu}} - {}^{\mathbf{u}}\mathbf{H_{uc}}\,{}^{\mathbf{u}}\mathbf{H_{cc}^{-1}}\,{}^{\mathbf{u}}\mathbf{H_{uc}})\,\mathbf{f_u}\,, \tag{8}$$

where $\mathbf{x_0}$ and $\mathbf{f_u}$ are the base displacement and the applied force, respectively. $\mathbf{x_0}$ is shown in Figure 3 and $\mathbf{f_u} = 0$ in this research. For the fatigue life-time estimation the stress response was predicted as:

$$\sigma = ({}^{\mathbf{u}}_{\sigma}\mathbf{H_c}\,{}^{\mathbf{u}}\mathbf{H_{cc}^{-1}})\,\mathbf{x_0}\,. \tag{9}$$

More details about the issues relenting to the extended SMURF method were presented in [29].

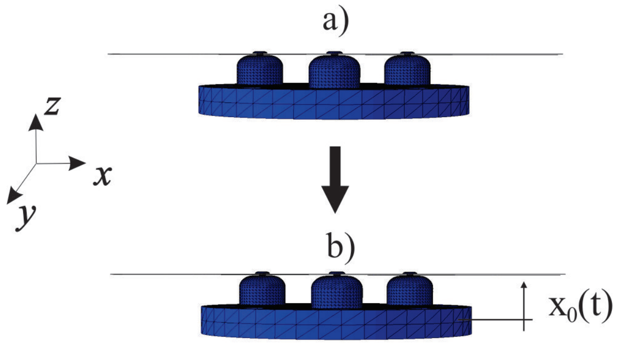

Figure 3: Simplified Dynamic Model; a) Unconstrained; b) Constrained

The estimated stress response (9) was used for the fatigue life-time estimation using the frequency-domain methods. Recently, Braccesi et al. [30] have made an overview of the strengths and weaknesses of the frequency approach with respect to the reference time-domain methods. Mršnik et al. [31] compared different frequency-domain counting methods and it turned out that the Tovo-Benasciutti method [32, 33] gave the best estimation in the majority of experiments, and hence it is used in this research. Instead of the time-until-failure, the damage intensity or the damage per unit time ($\overline{D}$) is used here:

$$\overline{D} = \nu_p\, C^{-1} \int_0^\infty s^k\, p_a(s)\mathrm{d}s\,, \tag{10}$$

where ν_p is the expected peak-occurrence frequency, calculated from the random process defined by the power spectral density, C and k are the material parameters and $p_a(s)$ is the cycle amplitude probability density function of the random process [31]. The fatigue life-times T are calculated from the damage intensity as:

$$T = \frac{1}{\overline{D}}\,. \tag{11}$$

With regards to Eq.(2), the damage accumulation is calculated as the ratio between the numbers of cycles. In this research the damage accumulation is calculated as the ratio between the estimated fatigue life-times (Eq. 11), and the damage-accumulation equation can be rewritten as:

$$D = 1 = \sum_{i=1} D_i = \sum_{i=1} \frac{t_i}{T_i}\,, \tag{12}$$

where t_i and T_i represent the time during which the structure is excited and the fatigue life-time for the i-th excitation level, respectively.

Initially, the linear numerical model was virtually excited for 30 minutes with a random acceleration excitation using a 5 g amplitude in the frequency range from 10 to 2000 Hz. The damage accumulation is calculated according to Eq.(12). For the numerical model the procedure is repeated at higher excitation

levels (6 g, 7 g,...), 30 min at each level, until the cumulative damage reaches 100 percent. The failure of the linear numerical model occurs after 15.83 min at the 12g excitation level.
Table 1 shows the details of the numerical results. The excitation levels, the fatigue life-times T_i, the excitation times t_i, the damage accumulation at each level $D[\%]$ and the accumulated damage $Acc.D[\%]$ are presented in the first, second, third, fourth and fifth columns, respectively.

Table 1. Linear Damage Accumulation for the Linear Numerical Model

Excitation level (i) [g]	T_i[min]	t_i[min]	$D[\%]$	$Acc.D[\%]$
5	1819.98	30	1.6	1.6
6	916.87	30	3.3	4.9
7	513.5	30	5.8	10.7
8	310.8	30	9.7	20.4
9	199.58	30	15	35.4
10	134.28	30	22.3	57.7
11	93.83	30	32	89.7
12	67.65	15.83	10.3	100

4 Experimental Research

The experimental setup is shown in Figure 4. It is used for the numerical model's validation and updating (Section 3), the estimation of the model's non-linearity and the identification of the experimental (real-model) fatigue life-times, which are compared with the numerical results.

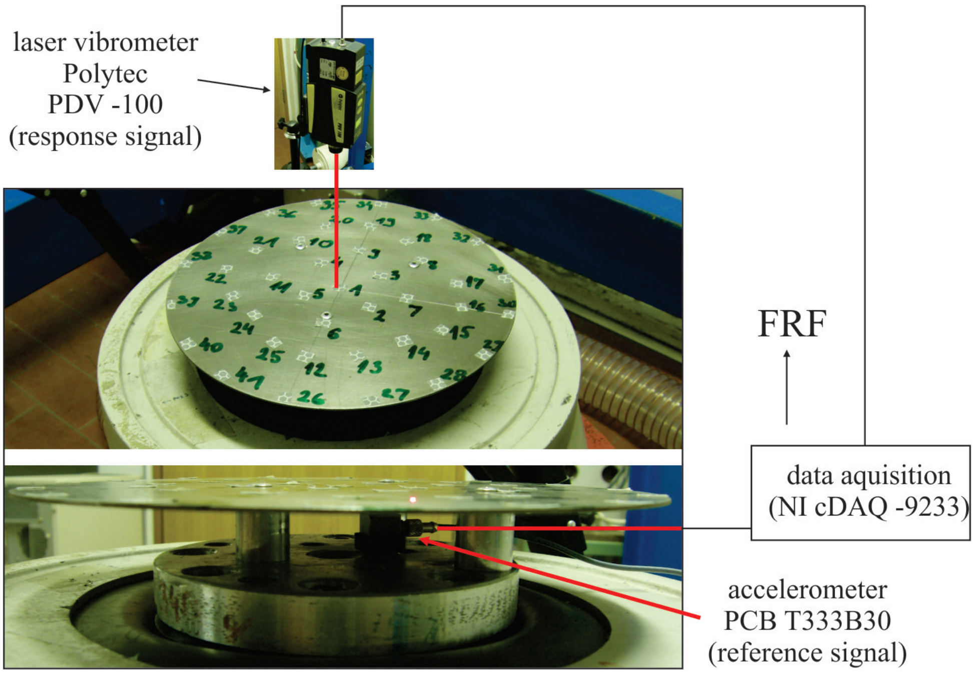

Figure 4: Electro-Dynamic Shaker and the Experimental Setup

Operational modal analysis was used for the experimental validation of the numerical model with a relatively low excitation level (1g RMS) where the structure was assumed to be linear. The experimental validation was performed by estimating the FRFs at 41 response points on the steel plate, see Figure 4.

The reference signal was measured on the aluminium supporting structure with an accelerometer (Figure 4). The numerical model was updated according to the experimentally obtained natural frequencies and the mode shapes. Figure 5 shows the numerically estimated and experimentally obtained frequency-response functions for one of the tested samples. The FRFs are estimated for point number 1 at the steel plate, see Figure 4.
The natural frequencies and the mode shapes are closely matched under 1400Hz. However, similar to the numerical mismatch, the experimental samples could also not be assembled to have an equal structural response in the higher frequency range. For this reason the research assumes that the frequency range below 1400 Hz has a significant influence on the fatigue.

As was explained in Section 2, estimated FRFs were used for the estimation of the model's non-linearity rates for different excitation levels, see Eq.(5). For the FRF estimation, the response was measured at the center of the attached steel plate and the reference signal was measured at the controlled shaker fixation head, see Figure 4.

Accelerated fatigue tests were performed with standard electro-dynamical shaker equipment. The vibration profile was controlled with a LDS Dactron vibration controller, a LDS PA1000L power amplifier and a personal computer. The hardware and software with a control loop were used to generate the excitation according to a given constant broad-band power spectral density in the range from 10 to 2000 Hz (the same excitation profile was used in the numerical research). At the same time, the measured signals were also discretized using a DAQ module and saved for later analysis. The sampling frequency was 10000Hz.

Experimental tests are also used for the identification of the experimental fatigue life-times by tracking the natural frequencies [17, 19, 21]. The frequencies between 500 Hz and 1400 Hz are tracked and if any of the natural frequencies changed by 10 Hz or more, the experiment was stopped due to fatigue damage.

4.1 Identification of damage at non-linear response

The corrected-linear damage-accumulation (3) was used with ten samples. Due to uncertainty in assembling those samples differ slightly and cause differences in the dynamical response and also rate of the non-linearity. The difference in assembling result primarily from the differences in the riveting force. The riveting force is hard to measure *in situ*; however, a separate research showed that the riveting force is close to 750 N and can differ up to ±10%. Further, the unilateral contact between the aluminium support structure and the steel sheet (see Fig. 2) primarily effect the non-linearities. Due to the above uncertainties the proposed estimation of the rate of non-linearity (5) is not sensitive to the changes in natural dynamics due to the riveting force.

Initially, each sample is excited for a short time at a 1g excitation level to obtain the linear FRF. After the initial test, seven samples were excited at levels of 5 g, 6 g, 7 g, 8 g,..., until failure (each level was applied for a maximum of 30 minutes). Additional three samples were excited in reverse excitation order (8 g, 7 g, 6 g...) until failure.

The linear damage accumulation for each excitation level was divided into thirty (one-minute-long) parts to calculate the corrected-linear, damage-accumulation D_c, according to Eq.(3):

$$D_c = \sum_j D_j \cdot R^{n_j} = 1 \,, \tag{13}$$

where the index j represents the number of steps (one minute represents one step for all the excitation levels) and D_i represents the linear damage accumulation for a particular step (the damage accumulation

for the whole period (for all the excitation levels) is divided so that the damage accumulation for each step is calculated). The rate of non-linearity n_i is calculated using Eq.(5).
The constant correction factor R is calculated as a result of minimizing the least-squares error between the numerical and experimental results for all seven samples (excited with increasing excitation levels):

$$E(R) = \sum_{j}^{m} [T_{ex_j} - T_{nu_j}]^2 \,, \tag{14}$$

$$\frac{\partial E}{\partial R} = 0 \,, \tag{15}$$

where T_{ex}, T_{nu} and m represent the experimentally identified, numerically (corrected-linear) estimated fatigue life-times, and the number of tested samples, respectively.

Increasing Level of Excitation. Seven samples are excited at excitation levels of 5 g, 6 g, 7 g, 8 g,..., until failure, 30 minutes at each level. The constant correction factor calculated according to Eq.(14) for all the samples is $R = 160$. A comparison of the numerical (corrected) and the experimental results is shown in Table 2. With the linear damage-accumulation (without correction) the damage occurs at the 12 g excitation level, which is significantly higher than experimentally observed, see Figure 6. With the identified $R = 160$, the corrected non-linear damage accumulation gives fatigue life-time results within 10% of the experimentally identified values, see Tables 2.
Figure 6 presents the numerically estimated damage-accumulation curves; linear damage-accumulation and corrected-linear damage accumulations for seven samples with different degrees of non-linearity. In the case of samples 1,5,6,7 the experimentally identified damage occurs before the numerically estimated damage, see Figure 6.

Table 2. Comparison of results

Sample	Numerical (corrected) [min], T_{nu}	Experimental [min], T_{ex}	Error [%]
1	80	82	2.43
2	92	100	8
3	97	108	10.18
4	107	110	2.77
5	79	77	2.6
6	54	51	5.9
7	85	93	8.6

Decreasing Level of Excitation. The additional three samples, the results for which will be presented here, are excited in reverse excitation order (8 g, 7 g, 6 g...) until failure. The idea is that the damage-accumulation is calculated in a similar way as for the samples excited with increasing excitation levels and with the same correction factor (R=160) from the same manufacturer. Sample 8 (see Figure 7) uses rivets from the same manufacturer as the previously investigated seven samples. The result of the damage accumulation for Sample 8 is estimated well (the numerically obtained fatigue life-time is close to the experimentally identified value, see Figure 7). Samples 9 and 10 apply rivets from the two additional rivet manufactures.
The numerical (corrected-linear) damage-accumulation results for samples 9 and 10 with the correction

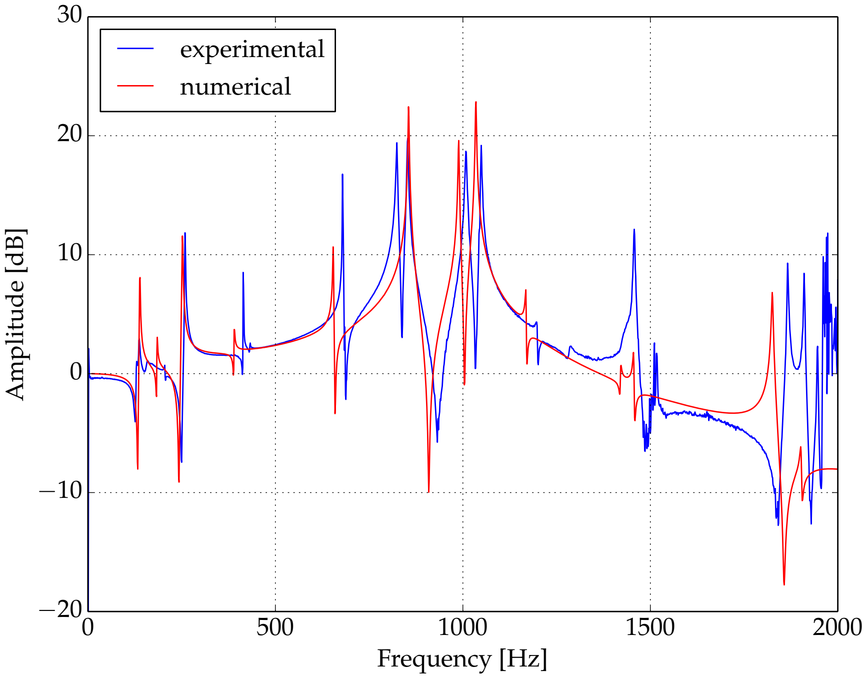

Figure 5: Numerical and Experimental FRFs

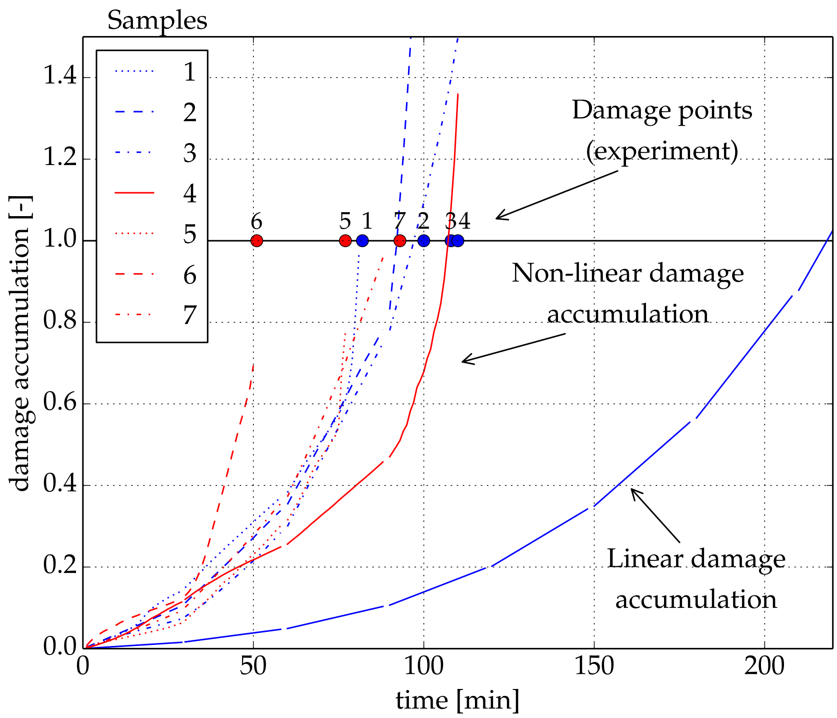

Figure 6: Linear and Corrected Damage Accumulation (R=160)

factor R=160 give fatigue life times that are shorter than experimentally identified (see Figure 7). These results are a consequence of the better material properties of the new rivets. Figure 8 shows the damage accumulation calculated with R=37 for sample 9 and R=14 for sample 10.

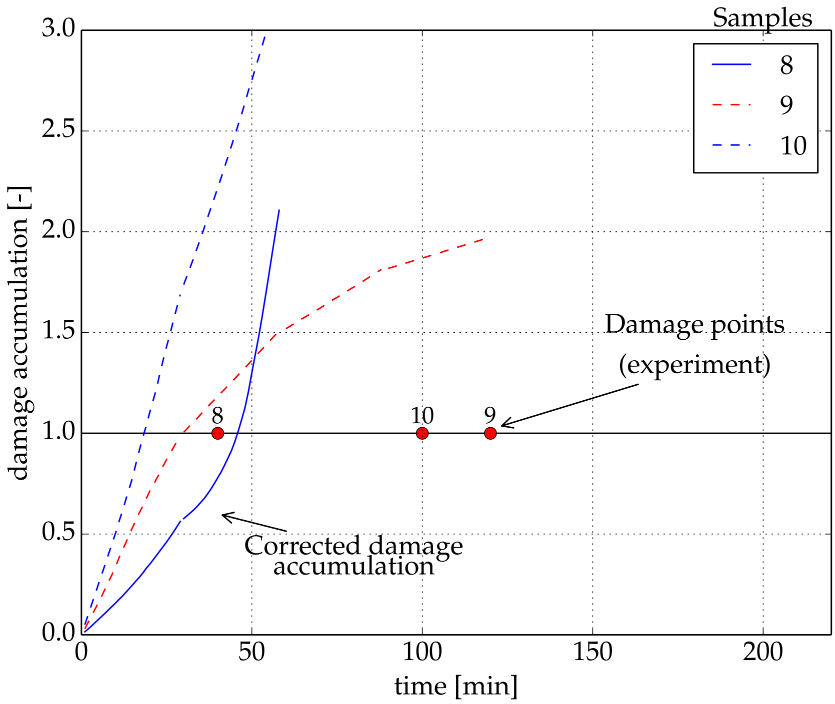

Figure 7: Corrected Damage Accumulation (R=160)

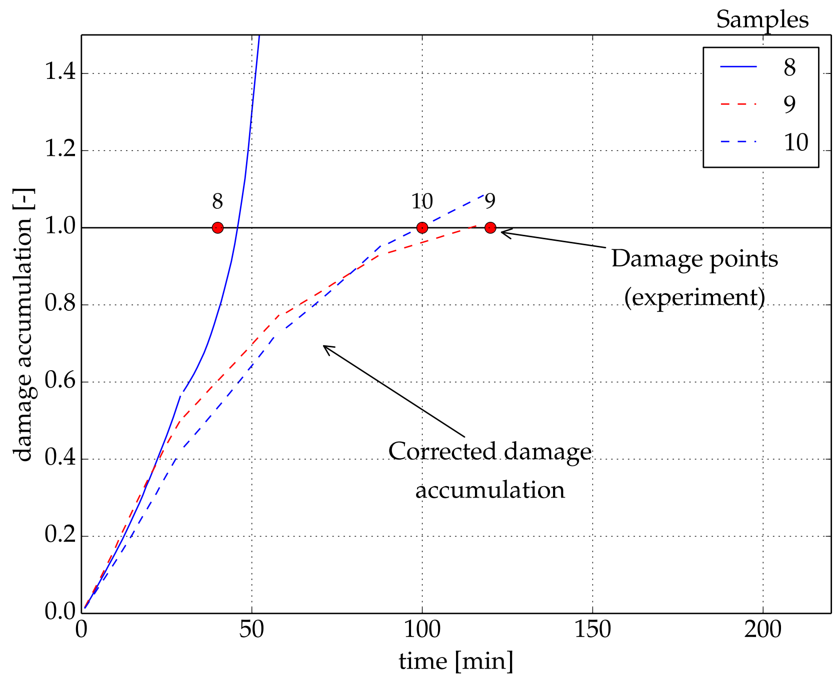

Figure 8: Corrected Damage Accumulation for Different Correction Factor R

5 Conclusions

For safer and lighter dynamically excited engineering structures, an appropriate damage-accumulation prediction is one of the most important issues. This research proposes a methodology for enhancing the numerically obtained linear damage-accumulation prediction with the experimentally identified structure's non-linearity. The proposed non-linearity identification in the frequency domain is relatively simple and can be made before the structure is exposed to the real fatigue loads. With the proposed correction of the linear damage-accumulation estimation also the linear structural-dynamics analysis can be extended to handle non-linearities.

This research opens up new possibilities for vibration fatigue damage estimation when dealing with structures exposed to broadband excitation and have a non-linear response. In application, the non-linear response is frequently observed at increased amplitudes of load. In this research, the total of ten samples are analyzed; seven samples have been tested with increased level of excitation and three with the decreased level of excitation. Rivets were of the same design, but from three different manufacturers. The FRFs did not differ significantly with different manufacturers; however the additionally identified constant R (required for the corrected linear damage-accumulation) was found to significantly differ with different manufacturers. Due to uncertainties (e.g. in assembling), the fatigue life-time of the samples with the rivets from the same manufacturer ranged from 51 to 110 minutes. However, with the experimentally identified constant R, the numerically obtained life-times were within $\pm 10\%$ of the experimentally identified. This was also true when the excitation level was decreasing. Further, the identified constant, required for the damage-accumulation correction, significantly differs with the rivet manufacturer.

Based on the finding of this research a better prediction of fatigue life is possible. This can result in more accurate prediction based maintenance and also more accurate numerical optimization with regards to fatigue damage at vibration loads.

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References

[1] J. Schijve, *Fatigue of Structures and Materials*, Delft University of Technology, Faculty of Aerospace Engineering, The Netherlands, 2009

[2] A. Fatemi and L.Yang, *Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials*, International Journal of Fatigue 20, 9-34, 1998

[3] A. Wöhler, *Über die Festigkeitsversuche mit Eisen und Stahl*, Zeitschrift für Bauwesen 20, 73-106, 1870

[4] A. Palmgren, *Lebensdauer von Kugellagern*, Zeitschrift des Vereins Deutscher Ingenieure 68, 339-341, 1924

[5] M. Miner, *Cumulative damage in fatigue*, Journal of Applied Mechanics - Transactions of the ASME 12, A159-A164, 1954

[6] A. Zambrano, D. Foti, *Damage indices evaluation for seismic resistant structures subjected to low-cycle fatigue phenomena*, International Journal of Mechanical Sciences 78, 106–117, 2014

[7] S. Marco, W. Starkey, *A concept of fatigue damage*, Transactions of the ASME 76, 627–32, 1954

[8] Y. Rabotnov, *Creep problems in structural members*, North Holland, Amsterdam, 1969

[9] L. Kachanov, *Introduction to continuum damage mechanics*, Martinus Nijhoff, Dordrecht, 1986

[10] J. Chaboche, *Continuum damage mechanics — a tool to describe phenomena before crack initiation*, Nuclear Engineering and Design 64, 233–47, 1981

[11] J. Lemaitre, J. Chaboche, *Mechanics of solid materials*, Cambridge University, UK, 1990

[12] V. Golub, *Non-linear One-dimensional Continuum Damage Theory*, International Journal of Mechanical Sciences 38, 1139–1150, 1996

[13] V. Dattoma, S. Giancane, R. Nobile, F. Panella, *Fatigue life prediction under variable loading based on a new non-linear continuum damage mechanics model*, International Journal of Fatigue 28, 89–95, 2006

[14] A. Aid, A. Amrouche, B. Bouiadjra, M. Benguediab, G. Mesmacque, *Fatigue life prediction under variable loading based on a new damage model*, Materials and Design 32, 183–191, 2011

[15] A. Djebli, A. Aid, M. Bendouba, A. Amrouche, M. Benguediab, N. Benseddiq, *A non-linear energy model of fatigue damage accumulation and its verification for Al-2024 aluminum alloy*, International Journal of Non-Linear Mechanics 51, 145–151, 2013

[16] P. Huffman, S. Beckman, *A non-linear damage accumulation fatigue model for predicting strain life at variable amplitude loadings based on constant amplitude fatigue data*, International Journal of Fatigue 48, 165–169, 2013

[17] D. Shang, M. Barkey, Y. Wang, T. C. Lim, *Effect of fatigue damage on the dynamic response frequency of spot-welded joints*, International Journal of Fatigue 25, 311–316, 2003

[18] D. Shang, *Measurement of fatigue damage based on the natural frequency for spot-welded joints*, Materials and Design 30, 1008–1013, 2009

[19] V. Dattoma, N. Giannoccaro, A. Messina and R. Nobile, *Prediction of residual fatigue life of aluminium foam through natural frequencies and damping shift*, Fatigue and Fracture of Engineering Materials and Structures 32, 601–616, 2009

[20] M. Česnik, J. Slavič, M. Boltežar, *Uninterrupted and accelerated vibrational fatigue testing with simultaneous monitoring of the natural frequency and damping*, Journal of Sound and Vibration 331, 5370–5382, 2012

[21] H. Hu, Y. Li, T. Suo, F. Zhao, Y. Miao, P. Xue, Q. Deng, *Fatigue behavior of aluminum stiffened plate subjected to random vibration loading*, Transactions of Nonferrous Metals Society of China 24, 1331-1336, 2014

[22] S. Benkabouche, H. Guechichi, A. Amrouche, M. Benkhettab, *A modified nonlinear fatigue damage accumulation model under multi axial variable amplitude loading*; International Journal of Mechanical Sciences 100, 180–194, 2015

[23] R. Owsinski, A. Nieslony, *Review of current state of knowledge on durability tests performed on electromagnetic shakers*, Modelling in Engineering 53, 117-123, 2014

[24] M. Česnik, *Strukturna dinamika pri vibracijskem utrujanju*; PhD Thesis, University of Ljubljana, Slovenia, 2013

[25] S. Suresh, *Fatigue of Materials*, Cambridge University, United Kingdom, 1998

[26] Y. Lee, J. Pan, R. B. Hathaway, M. Barkey, *Fatigue Testing and Analysis; Theory and Practice*, Elsevier, Oxford, 2005

[27] O. Basquin, *The exponential law of endurance tests*, Proceedings of American Society of Testing Materials 10, 625-630, 1910

[28] N. Maia, J. Silva, *Theoretical and Experimental Modal Analysis*, Instituto Superior Technico, Portugal 1997

[29] M. Česnik, J. Slavič, P. Čermelj, M. Boltežar, *Frequency-based structural modification for the case of base excitation*, Journal of Sound and Vibration 332, 5029–5039, 2013

[30] C. Braccesi, F. Cianetti , G. Lori, D. Pioli, *Random multiaxial fatigue: A comparative analysis among selected frequency and time domain fatigue evaluation methods*, International Journal of Fatigue 74, 107–118, 2015

[31] M. Mršnik, J. Slavič and M. Boltežar, *Frequency-domain methods for a vibration-fatigue-life estimation - Application to real data*, International Journal of Fatigue 47, 8-17, 2013

[32] R. Tovo, *Cycle distribution and fatigue damage under broad-band random loading*, International Journal of Fatigue 24, 1137-1147, 2002

[33] D. Benasciutti and R. Tovo, *Spectral methods for lifetime prediction under wide-band stationary random processes*, International Journal of Fatigue 27, 867-877, 2005.