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Related to the Structural Vibrations

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The ac magnetostriction in electrical steel is commonly characterized in the time domain (e.g., the peak-to-peak, zero-to-peak amplitude) and also in the frequency domain (e.g., a harmonic analysis). However, due to the dynamical coupling of the test sample with the experimental set-up, the characterization of the magnetostriction (especially the one in the frequency domain) can give the wrong result. This research focuses on an experimental frequency characterization of magnetostriction and gives the theoretical background of the test sample's dynamical coupling with the experimental set-up. The discussed natural dynamics of the test sample from the point of view of the different boundary conditions that can be used at the experiment gives a clear picture of the dynamical coupling. Besides the theoretical background, a detailed experimental approach is presented. This research theoretically and experimentally showed that the dynamical coupling of the sample can result in incorrect characterization of the magnetostriction. However, with the theoretical guidelines presented, the dynamical coupling can be completely avoided, which results in an accurate characterization of the magnetostriction.

Index Terms—Frequency domain analysis, Electrical steel, Experiment, Magnetostriction, Structural vibrations.

I. INTRODUCTION

A s the harshness of noise is becoming increasingly important to the quality of life, the noise of electrical devices is also getting more and more important. Several studies found the magnetostriction of electrical steel components under ac magnetization to be one of the major causes of noise emissions from such devices [1], [2], [3]. This research deals with experimental frequency characterization of the magnetostriction phenomena as one of the root sources (besides, e.g., the imbalance in rotating machines) of noise in electrical devices. Since magnetostrictive materials are commonly used in a wide range of applications (e.g., the transformer cores, stators, and rotors of specific electric motors), a broad technical field is closely linked to this topic.

Research on magnetostriction frequently focuses on the time-domain properties (e.g., the peak-to-peak, zero-to-peak amplitude) [4], [5]. However, when considering noise generation, a time-domain analysis exhibits significant drawbacks. One of the first researchers to study the frequency content of magnetostriction was Reiplinger [6], who found that the peak-to-peak amplitude is not able to take into account the harmonic components of magnetostriction and therefore fails to relate to the noise generation of electrical steel. Since the sound pressure in the fluid surrounding a vibrating object is proportional to the velocity rather than to the displacement of the object [7], higher frequency vibrations are more effective at generating sound. The research [6] revealed an agreement between the A-weighted magnetostriction velocity level and the A-weighted sound pressure level [8]. While such an agreement can be achieved on mechanically simple systems, like the featured single-sheet tester, the relations are, in general, not so straightforward. A significant influence of the transformer-core assembly techniques and the clamping pressure on the noise level and the noise spectrum was found by Snell [9]. Evidently, the mechanical properties of a structure have a significant influence on the resultant vibrations and noise. Especially when complex mechanical joints [10], [11] and friction-related non-linearities [12] are present in the structure, the frequency content of the response is highly variable. The research by Snell showed various magnetostriction harmonics can prove critical in terms of noise generation, thus confirming the significance of the frequency characterization of the magnetostriction phenomenon.

The research by Reiplinger [6] was among the first to focus on the harmonics of magnetostriction in electrical steel. It disclosed one of the key properties of harmonics, i.e., the proportional increase of higher harmonics with respect to the fundamental when nearing magnetic saturation. As a consequence, the noise level of the tested sheet sample was found to increase, despite the constant level of the peak-to-peak value. In addition to Reiplinger, other researchers also focused on the frequency characterization of magnetostriction in electrical steel [1], [13], [14]. One fundamental study was performed by Mapps

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and White [13], who investigated the magnetostriction of a single grain of electrical steel. They observed a substantial increase in the first three harmonics when the sample was exposed to compressive stress. Additionally, a ratio of -1/2 between the transverse and the longitudinal magnetostriction harmonics was found.

Several researches in the frequency domain also considered the coupling mechanisms between the harmonic components in the magnetising flux and those in the magnetostrictive strain [2], [15], [16]. The influence of the phase difference between the fundamental flux component and its third harmonic was researched by Mapps and White [15]. Compared to sinusoidal induction, the second and third magnetostriction harmonics are larger only with the onset of magnetic saturation, when the phase difference reduces. Kim et al. [2] presented a theoretical model to describe the symmetrical nature of magnetostriction, where excitation with odd flux harmonics results in the response of even magnetostriction harmonics. In contrast to natural induction, used in [2], the effect of artificially included magnetising harmonics was considered by Anderson [16]. For the specific conditions of the research a theoretical estimation of the dominant magnetostriction harmonic was presented.

In the time domain, the magnetostriction is measured in the range of microstrain ($\mu\epsilon$), while in the frequency domain, the amplitudes of the harmonics are in the range of a few fractions of microstrain. Consequently, the frequency-domain characterization requires an experimental set-up with higher sensitivity and a wider dynamical range. Besides the magnetostriction amplitude, the measurement sensorics need to match the frequency content of the analyzed phenomenon. Furthermore, as the measured sample can dynamically interact with the experimental set-up (i.e. it is dynamically coupled), the mechanical design of the set-up is of utmost importance. In detail, electrical steel sheets are relatively thin and therefore vibrations of the test samples in the in-plane direction and (especially) in the out-of-plane direction are likely. This is most pronounced when the excitation frequency (or frequencies) coincide with the natural frequency (or frequencies) of the set-up and the mechanical resonance occurs. Phway and Moses [4] researched magnetostriction with regards to the in-plane dynamics of the test sample and found that even a small excitation can cause a significant response. The research by Ghalamestani et al. [17] presents out-of-plane vibrations to considerably lower the repeatability and the quality of the measurement. Similarly, in the study by Kim et al. [2], the measurement of magnetostriction was restricted to unrealistically low frequencies (the order of Hz) due to a distortion in the mechanical motion. To some extent, unwanted vibrations can be cancelled out with the use of a reference signal, e.g., Mapps and White [15] presented a method of cancellation for signal components due to perpendicular and transverse vibrations. Furthermore, surrounding vibrations are a known cause of measurement errors [14], [18]; however, these can be significantly reduced with a proper vibro-isolation between the experimental set-up and its surroundings [18].

While previous research was focused on the identification of in- and out-of-plane vibrations with regard to errors in the magnetostriction measurement, this research is focused on decreasing/eliminating the source of these errors. The theoretical and experimental research presented here discusses the dynamical coupling of the test sample with the experimental set-up and the effect dynamical coupling presents to the measurement. The capability of the set-up was demonstrated on a specimen of grain-oriented electrical steel.

This manuscript is organized as follows. Section II gives the theoretical background on the dynamics of the test sample. Section III compares the Epstein test sample with a new test sample, introduced in this study. The experimental set-up is discussed in the Section IV. Section V presents the measurement results. The conclusions are given in the last Section.

II. THEORETICAL BACKGROUND

Experimental research on ac magnetostriction focuses on the detection of in-plane deformations of electrical steel, caused by an ac magnetic field. However, the experimental set-up is a dynamical system and the in-plane dynamics depends on the excitation mechanism (via magnetostriction) as well as on the in- and out-of-plane natural dynamics of the system. The in- and out-of-plane dynamics must therefore be analyzed to ensure that the measurement results are intrinsic to the test sample and not influenced by the experimental set-up.

A. Natural dynamics of the experimental set-up

An analysis of the natural dynamics of the experimental set-up is the first step when evaluating an experimental set-up with regards to vibrations. To avoid dynamical coupling of the sample with the experimental set-up, the frequency range of interest should, in general, not be within the range of its natural dynamics. In the following, different dynamical models of the experimental set-up will be considered.

1) Sample with free-free boundary conditions

In case of a rectangular-shape sample, the first in-plane natural frequency is [19]:

$$f_{I,FF,1} = \frac{1}{2L} \sqrt{\frac{E}{\rho}},\tag{1}$$

where L is the length of the sample, E is the modulus of elasticity and ρ is the density of the material. As it is evident from (1), the width w and the thickness h of the sample do not influence the in-plane natural frequency.

In the presented configuration, the first out-of-plane natural frequency is [20]:

$$f_{O,FF,1} = 3.563 \frac{1}{L^2} \sqrt{\frac{D}{\rho h}}$$
(2)

where D is the flexural rigidity of the sample:

$$D = \frac{Eh^3}{12\left(1 - v^2\right)} \tag{3}$$

where ν is Poisson's ratio.

2) Sample with clamped-free boundary conditions

In experimental set-ups, the positioning of the sample is often maintained by clamping one end of the sample while leaving the other end free to move. In this case, the first in-plane [19] and the first out-of-plane [20] natural frequencies are:

$$f_{I,CF,1} = \frac{1}{4L} \sqrt{\frac{E}{\rho}},\tag{4}$$

$$f_{o,CF,1} = 0.560 \frac{1}{L^2} \sqrt{\frac{D}{\rho h}}$$
(5)

3) Sample that is clamped at one end with a translational mass at the other

The dynamical model of the sample that is clamped at one end with a translational mass at the other is shown in Fig. 1. The translational mass m is the mass of the linear guides, frequently used in experimental set-ups for magnetostriction measurements to ensure one-directional movement. Friction of the guides is not regarded in this model.

The analytical solution for the first in-plane natural frequency can be achieved under an assumption of linear displacement along the length of the sample [21]:

$$f_{I,CTM,1} = \frac{1}{2\pi} \sqrt{\frac{AE}{Lm_e}}$$
(6)

where A = hw is the cross-section of the test sample and $m_e = m + \rho AL/3$ is the equivalent dynamical mass.

The first out-of-plane natural frequency is equal to that of the clamped-clamped (and free-free) boundary conditions [20]:

$$f_{o,CTM,1} = 3.563 \frac{1}{L^2} \sqrt{\frac{D}{\rho h}}$$
(7)



Fig. 1. Sample that is clamped at one end with a translational mass at the other.

B. Forced dynamics of the experimental set-up

1) In-plane forced dynamics

As emphasized in the Introduction, the magnetostrictive response depends on the dynamical coupling between the test sample and the experimental set-up. The experimental set-up is dynamically similar to the model, shown in Fig. 1. As the excitation with the magnetostrictive force (i.e., the mechanical force, which produces the same deformation as the magnetic field [22], [23]) F_{MS} is in the in-plane direction, the model shown in Fig. 2 will be discussed in detail. For this research, the magnetostrictive force was assumed to be sinusoidal: $F_{MS0}(t) = F_{MS0} \sin(2\pi ft)$, where F_{MS0} is the force amplitude, f is the excitation frequency and t is the time.

For harmonically excited linear systems, the response is also harmonic and the response amplitude X is relatively easily determined [21]:

$$X = \frac{F_{MS0}}{k}\beta$$
(8)

where k is the in-plane stiffness of the sample (k = AE/L) and β is the dynamic magnification factor (assuming an undamped system):

$$\beta = \frac{1}{1 - \left(\frac{f}{f_0}\right)^2} \tag{9}$$

with f_0 being the natural frequency of the system

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m_e}} \tag{10}$$



Fig. 2. Dynamical model of the in-plane forced dynamics.

The dynamic magnification factor β shows that the response amplitude increases significantly at resonance, see Fig. 3. For example, a sinusoidal force with $f/f_0 = 0.22$ produces a β of 1.05, which is a 5% increase in the response. This effect presents a disturbance in the measurement of magnetostriction as it artificially scales the response. The relative error can be calculated as

$$\eta = \operatorname{abs}(\operatorname{abs}(\beta) - 1) \cdot 100 \quad [\%].$$
⁽¹¹⁾

The absolute value of the dynamic magnification factor β and the relative error η are shown in Fig. 3.

Like was deduced for the in-plane direction, one can deduce the out-of-plane direction equations and the same conclusions about the response close to the resonance could be made.

Due to the dynamical coupling, the in- and out-of-plane dynamics interact and if the response due to the magnetostrictive force is close to any of the natural frequencies, this will result in both of the directions. The level of this cross-sensitivity is not the focus of this research and relates among others to the geometry and the geometrical and the material imperfections.



Fig. 3. The absolute value of the dynamic magnification factor β and the relative error η as function of f/f_0 .

III. EPSTEIN SAMPLE VS. NEW SAMPLE IN THE FREQUENCY DOMAIN

With regards to the theoretical background, the Epstein test sample [24] and a newly proposed test sample will be compared here. The Epstein test sample was primarily designed for the measurement of the magnetic properties of a material, but it is often utilized in research on magnetostriction as well [1], [24]. An Epstein test sample measures 305 mm in length and 30 mm in width, while the thickness is equal to that of a single sheet (single-sheet tester). The newly introduced test sample is designed to meet the dynamical criteria for the performed research. It is assembled as a stack of 10 sheets with a measurement region of 40 mm x 40 mm, Fig. 4.



Fig. 4. New test sample.

The mechanical assembly of the experimental set-up is analogous to the model depicted in Fig. 1. The sample is clamped at one end (clamped support) and fixed onto a precision linear table at the other (translational mass m). The linear table enables an in-plane displacement with minimal friction. It is made from aluminium to minimize the magnetic and inertial effects, (6).

The natural frequencies of the two types of test samples were calculated for several cases of boundary conditions; the material and the set-up parameters are taken for the performed measurement: v = 0.3, $\rho = 7650 \text{ kg/m}^3$, $E = 1.08 \cdot 10^{11} \text{ Pa}$ and m = 0.81 kg. For the Epstein sample, the used parameters h = 0.27 mm, L = 305 mm and w = 30 mm correspond to a single test sheet of 0.27 mm thickness (thickness of the measured material) under the assumption that the measurement length spans the whole length of the sample.

The new sample consists of a stack of 10 steel sheets (Fig. 4), and for analytical model equations (1) to (7) further simplifications are required. For the in-plane dynamics, the stacked sample can be assumed to be solid as the stiffness is simply summed up. Accordingly, h = 2.7 mm, L = 40 mm and w = 40 mm. The out-of-plane dynamics depends on the interaction between the sheets of the stack; a reasonable assumption here is that there is no interaction between the sheets and this means that the natural frequency of the whole stack is theoretically the same as the natural frequency of one sheet only. However, in reality, there are interactions, which lead to a stiffer dynamical system and therefore increase the out-of-plane natural frequency. This means that the assumption of no interaction is on the safe side of our analysis (the experimental section will show the

accuracy of this approach). For the out-of-plane dynamics the (one steel sheet) parameters are: h = 0.27 mm, L = 40 mm, and w = 40 mm.

Table I presents the theoretical natural frequencies of the samples for three types of boundary conditions. Additionally, the measured values for the new sample are given.

TABLE I				
NATURAL FREQUENCIES OF THE TEST SAMPLES				
	First in-plane freq.		First out-of-plane freq.	
	Epstein sample	New sample	Epstein sample	New sample
Free-free	6160 Hz	46 967 Hz	12 Hz	684 Hz
Clamped-free	3080 Hz	23 483 Hz	2 Hz	107 Hz
Clamped- translational mass	298 Hz	2999 Hz	12 Hz	684 Hz
Clamped- translational mass (measured)	/	2110 Hz	/	766 Hz

An ac magnetic field of frequency f results in pronounced even magnetostriction harmonics of 2f (fundamental harmonic), 4f, and 6f, while the higher frequency content is less important [1], [14]. In the performed magnetostriction characterization, the excitation frequency was f = 50 Hz and the harmonics of 100, 200, and 300 Hz have been researched.

The results show that the Epstein sample does not meet the dynamic requirements of the measurement in any of the considered cases. In the case of free-free or clamped-free sample, the out-of-plane frequencies are estimated at 12 and 2 Hz, respectively. If fixed as shown in Fig. 1 (clamped-translational mass), the sample is expected to have the first in- and out-of-plane natural frequencies, theoretically, at 298 Hz and 12 Hz, respectively. As the presented natural frequencies occur below 300 Hz, it is reasonable to expect that the magnetostrictive harmonics of 100, 200, and 300 Hz can be distorted by the first or any of the following natural modes in this frequency range.

Under clamped-translational mass boundary conditions, the proposed test sample proves suited to the measurement. In contrast to the Epstein sample, the exhibited natural frequencies are significantly above the frequency range of interest. More details on the proposed sample's natural dynamics and the influence on the magnetostriction measurement are discussed in the following experimental section.

IV. EXPERIMENTAL SET-UP

The experimental set-up for measuring the magnetostriction is schematically shown in Fig. 5 and the real set-up (mechanical assembly) is shown in Fig. 6.

Since high-permeability grain-oriented electrical steel can feature grains of 20 mm and more in diameter [25], it is important that an ample volume of material is included in the measurement. As already discussed, the proposed test sample is made out of 10 steel sheets (180 mm x 40 mm) and the measurement region is 40 mm x 40 mm. The measurement volume of the new sample is comparable to the values reported in [14], [26], and exceeds those quoted in [17], [24].

The measurement is performed with a PC running a custom-designed measurement application. The magnetisation signal is generated with a 16 bit analog signal generator, amplified with a 1 kW laboratory power amplifier and applied to the magnetising coil. The coil (550 turns) is positioned on the yoke, which, together with the sample, forms a closed path for the magnetic flux. The yoke is made as a glued stack of electrical steel laminations. The gap between the yoke and the test sample is held constant using a non-magnetic spacer, providing a more uniform flux distribution. The magnetic flux density in the sample is monitored with a search coil (4 turns) that is tightly wound around the sample to minimize the influence of the air flux. A 16 bit analog input module, running at 5 kHz sampling frequency, is used to acquire the signal of the search coil. The reading is used in a feedback loop that is integrated into the measurement application, ensuring the required magnetic flux density takes place in the sample. The demagnetization of the test sample and the yoke is performed before every measurement.

The magnetostriction is measured through the movement of the linear table using two piezoelectric accelerometers (sensitivity 2 pC/(m/s^2)). One accelerometer is attached to the clamped support and the other to the linear table. The sensors are connected to a high quality charge amplifier. The amplified signals are acquired using a 24 bit analog input module, running at 5 kHz sampling frequency. The relative acceleration is obtained by subtraction in the time domain. The relative acceleration is then filtered with a high-pass filter at 40 Hz, followed by double integration to obtain the relative displacement. The relative displacement length to obtain the magnetostrictive strain. Finally, a Fourier transform is used to obtain the amplitude spectra of the magnetostriction.

An anti-vibration table is used to minimize the dynamical coupling to the environment. Furthermore, precise positioning of the sample and consistency in applying the clamping force are required for repeatable measurements. Regarding the peak-to-peak value, the repeatability of the measurements at 50 Hz and 1.7 T (single sample consecutively remounted) was found to be within \pm 5% of the mean.



Fig. 6. Mechanical assembly of the experimental set-up.

A. Experimental analysis of the dynamics of the set-up

The dynamic behaviour of the set-up was already theoretically analyzed and here an experimental verification is made.

1) In-plane natural dynamics

The in-plane natural dynamics was excited with an impact excitation of the linear table in the in-plane direction. An excitation ball was used to excite a broad frequency range including the investigated natural modes. The response of the system was measured using the accelerometers of the set-up, coupled to the existent charge amplifier and the data acquisition system. The frequency power spectrum of the acceleration of the linear table relative to the clamped support was used to determine the natural frequencies. The first was measured at 2110 Hz, which differs from the theoretical value of 2999 Hz. Because the clamped support in the experiment is not ideally rigid, as assumed in the analytical deductions, a lower value was expected. For the sake of briefness and so as not to lose focus, detailed results are omitted here.

2) Out-of-plane natural dynamics

To measure the out-of-plane natural frequencies, the impact excitation of the sample was made in the out-of-plane direction. An excitation hammer was used to excite the investigated natural modes and a laser Doppler velocimeter (sensitivity 0.2 V/(mm/s)) was used to measure the out-of-plane response of the sample. Acquisition of the signal was performed using a 24 bit analog input module, running at 5 kHz sampling frequency. Natural frequencies were determined from the vibration velocity power spectrum. The first out-of-plane natural frequency was measured at 766 Hz, while the theoretical value is at 684 Hz. As the theoretical model assumes no interaction between the stacked steel sheets, the results are reasonable.

V. MEASUREMENT AND RESULTS

A. Magnetostriction characterization

The measurement was carried out on a specimen of high-permeability grain-oriented electrical steel, designated according to the standard EN 10107:2005 as M 90-27 P. Three test samples were prepared with their rolling direction parallel to the magnetizing (and measurement) direction of the set-up. The samples were excited with a sinusoidal magnetic field of 50-Hz frequency and a 1.7-T amplitude of the magnetic flux density.

The peak-to-peak amplitudes of the observed magnetostriction waveforms are presented in Table II, see also Fig. 7. The agreement of the peak-to-peak values between the samples was found to be within \pm 3% of the mean.

The frequency-domain analysis showed that the harmonics of 100, 200, and 300 Hz for different samples are in agreement, within \pm 8% of the mean. The mean values of the harmonics are presented in Fig. 8: the 100-Hz harmonic is the dominant component, with the 200-Hz harmonic exhibiting 18% of its amplitude and the 300-Hz harmonic only 4%. The disclosed hierarchy is in good agreement with the findings of Mapps and White [13] and Hirano et al. [14].





Fig. 7. Magnetostriction waveforms of the three test samples.



B. Magnetostriction close to the out-of-plane resonance

The experimental set-up demonstrated non-resonant behaviour in the range up to 300 Hz. However, to show that out-of-plane dynamics can significantly distort the in-plane dynamics and therefore the magnetostriction, the samples measured in Section 5.A were analyzed again under altered experimental conditions.

The measurement region of the sample was increased to a length of 64 mm (the width is still 40 mm), decreasing the first natural frequency to 314 Hz (experiment). The magnetic excitation was applied at frequencies ranging from 50 to 200 Hz with the step of 25 Hz. The amplitude of the magnetic flux density was 1.0 T. Additionally, a control measurement was performed, employing a sample with a 40-mm measurement length while all the other parameters were the same. The focus was on the fundamental out-of-plane and in-plane harmonics. Out-of-plane dynamics of the sample was measured with the laser Doppler velocimeter, featured in Section IV.A.2. Fundamental displacement harmonic was obtained from the velocity signal by a single integration and a subsequent Fourier transform. In-plane magnetostriction was measured with the technique presented in Section IV. For the three analyzed samples, mean values of the identified out-of-plane and in-plane harmonics were calculated at each measurement point.

Amplitude of the out-of-plane fundamental harmonic is presented in Fig. 9. In the case of the sample with a 64-mm measurement length, the first out-of-plane natural frequency occurs inside the range of the mechanical excitation via magnetostriction. Accordingly, the out-of-plane response is significantly increased close to this frequency, at 300, 350, and 400 Hz. In contrast, the control sample exhibits no significant out-of-plane vibrations in the analyzed frequency range (the measured out-of-plane resonance is at 766 Hz).

Fig. 10 presents the fundamental harmonic of the measured in-plane magnetostriction. At 100 Hz both samples are far from the out-of-plane natural frequency and the measured magnetostriction at both samples is comparable (0.10 and 0.11 μ c for the short and the long sample, respectively). However, above 200 Hz, when the longer sample approaches the first out-of-plane natural frequency, the magnetostriction results start to disagree.



Fig. 9. Amplitude of the out-of-plane fundamental harmonic.



VI. CONCLUSIONS

This theoretical and experimental research on the frequency-domain magnetostriction identification focuses on the structural dynamics of the experimental set-up as it interacts with the test sample. The simplified theoretical models have been shown to describe, relatively well, the experimentally measured natural frequencies in the in- and out-of-plane directions.

The theoretical section gives the background for a proper experimental set-up design to avoid any dynamical coupling of the in- and out-of-plane dynamics.

The detrimental effect of the dynamical coupling on the measurement was demonstrated experimentally. A systematic research on a series of excitation frequencies showed accurate magnetostriction measurement is not possible, when mechanical resonance is present.

The introduced measurement method was shown to be appropriate for the magnetostriction frequency characterization up to approximately 50% of the first out-of-plane natural frequency of the sample.

The proposed test sample and method do not require any bracing elements, e.g., a single support plate [26], a combination of two support plates [27] or a former [24]. This additionally contributes to the quality of the measurement, since the disturbing friction between the support and the sample cannot occur.

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