On multibody-system equilibrium-point selection during joint-parameter identification: A numerical and experimental analysis

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Abstract

Computational simulations of a multibody dynamic response are an important tool for the analysis and design of various mechanical systems. While the governing dynamic equations of these systems are well known, the identification of model parameters, especially those associated with joints, can prove difficult and time consuming. Traditionally, experimental methods are used to deduce the physical joint parameters by isolating the joint from the rest of the structure and testing it under static or dynamic loads. An alternative to pure experimental joint-parameter identification is the model-based methods, which rely on finding such parameter values that the predicted dynamic response coincides with that of the real system. As the equations of multibody systems are highly nonlinear, linearization techniques are applied to efficiently deduce the system's dynamic parameters using modal analysis. Although significant progress has been made in recent years, none of the studies that propose the linearization technique has addressed the effect of multibody system equilibrium-point selection on the accuracy of the parameter-identification procedure. Therefore, here, a new general model-based parameter-estimation method is proposed that minimizes the difference between the experimentally and numerically obtained dynamic system's natural frequencies. The basic idea of the proposed method relies on the development of an algorithm that identifies the optimal equilibrium point of the linearization for a given multibody system. The equilibrium point is deduced in such a way as to minimize the interplay between the different joint parameters on the system's natural frequencies. Using the proposed approach it is possible to localize the influence of the individual joint's stiffness parameters to one particular natural frequency. The presented case study highlights the efficiency of the developed parameter-estimation procedure and with this the importance of a proper linearization equilibrium-point selection for a reliable and accurate parameter-identification process.

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1. Introduction

Due to advancements in the power of computers, computational simulations of a multibody system's dynamic response have become an important tool for the design and analysis of physical systems. Real mechanical systems usually consist of many components that are connected together though various joints. The mathematical formulations governing the dynamics of a multibody system are generally known and can be obtained from numeric or symbolic algorithms [1]. Although the governing equations are well defined, the modeling of such systems still represents a challenge due to the uncertainty in the system's parameters, especially those associated with the joints. The identification of a joint's stiffness and damping properties [2, 3, 4] is therefore of great importance for the development of validated numerical models.

Traditional experimental methods deduce the physical joint's parameters by isolating the joint from the rest of the structure and testing it under static or dynamic loads [5, 6]. The removed joint is installed in the test apparatus using additional joints that can affect the quality of the measurement results. The testing conditions such as loads, pre-loads, temperature, surface mating, position, etc., also determine the joint's physical characteristics [7] and are sometimes hard to reproduce in a laboratory. As modern materials are frequently nonlinear in nature, the test-obtained joint parameters might also not be applicable for the end use of the modeled product. Moreover, it is practically impossible to isolate the joint in certain cases, e.g., in biomechanical systems, where we also have to account for the elastic and nonelastic power that the muscle systems set against the externally induced motion of the joint [8, 9, 10].

Alternatives to the pure experimental joint-parameter identification are the model-based methods, which are typically a combination of experimental data and the results obtained from numerical models. The identification problem focuses on finding such parameter values that the predicted dynamic response coincides with that of the real system [11, 12]. The cost function is defined in terms of an output error, which leads to a maximum-likelihood estimation. The equations describing the dynamic response of multibody systems are usually highly nonlinear. This is due to the geometrical nonlinearities associated with rigid-body rotations, which makes the problem of parameter identification particularly challenging [13]. The majority of the contributions devoted to this subject have focused on some specific applications, such as robotics [14, 15, 16, 17] or biomechanical systems [18, 19, 20]. In robotics, motors are usually placed at the joints for control and the joint parameters (stiffness, damping, ...) are obtained by comparing the robot dynamic model with either the measured motion (usually at the end-effector) [16, 21] or the motor force/torque [22, 23]. Researchers in [24] used springs to model the transmissions between motors and the rigid links in flexible-joint robots. This approach resulted in supplementary degrees of freedom in comparison to rigid body robot modeling. A detailed joint model was used in [23], which included backlash, friction on motor and arm side, damping and nonlinear stiffness. A frequency-domain joint model parameter identification was also utilized, e.g. [24] used only motor-side measurements to obtain the system frequency response functions.

Compared to the area of robotics, relatively little attention has been paid to joint-parameter estimation in the general area of multibody systems [25, 26, 27]. The presence of nonlinearities greatly complicates the parameter-identification process since the linear-superposition principle becomes inapplicable and therefore an explicit time integration must be performed to obtain the system's dynamic response. A time-domain procedure for the parameter identification of a nonlinear, multi-degree-of-freedom system was presented in [28]. In [13] the authors proposed a Lie-series technique to estimate the parameters of general multibody systems. In order to achieve a more effective and less time-consuming parameter-optimization process it is often desirable to linearize the equations of motion. The researchers in [29] developed a parameter-identification scheme for linear or linearized systems that are limited to the modeling of torsional vibrations in mechanical systems like rotors and power trains. Researchers have also presented a method that rewrites the equations of motion into a linear form with regards to the parameters that are to be identified [30, 31]. The method is broadly used in the field of robotics for calibration and control and is therefore intended for fast, real-time calculations. The robot dynamic model parameters are identified using the standard least squares techniques by comparing the calculated and measured joint torques, positions, velocities and accelerations [32, 33].

Although significant progress has been made in recent years, none of the above studies has addressed the effect of multibody-system equilibrium-point selection on the accuracy of the parameter-identification algorithm. Therefore, in our study, a new general model-based parameter-estimation algorithm is proposed that relies on the optimal system's equilibrium-point selection during the joint-parameter identification process. The method is based on minimizing the difference between the experimentally and numerically obtained dynamic system's natural frequencies. The comparison of the natural frequencies is used as they are highly sensitive to any change of the system's parameters and can also be measured with a high degree of accuracy [34]. As the equations of multibody systems are highly nonlinear, the symbolic linearization technique is applied to extract the modal parameters. While there are a number of linearization techniques used to calculate the time responses for a specific set of initial conditions [35, 1], only limited work has been done to obtain local linearizations of multibody systems at equilibrium points [36, 37]. In our paper the method presented in [37] and upgraded in [35] is utilized because it enables the linearization of a general multibody system at any selected equilibrium point. The basic idea of the proposed parameter-identification procedure relies on the implementation of an algorithm that deduces the optimal equilibrium point for the linearization of a given multibody system. The equilibrium point is chosen in such a way as to minimize the interplay between different joints for the system's natural frequencies. Using the proposed approach it is possible to localize the influence of an individual joint's stiffness parameters to only one selected natural frequency. This results in an accurate and efficient procedure that makes it possible to exploit all the advantages of a on-line parameter estimation.

The efficiency of the proposed parameter-estimation algorithm is demonstrated on a numerical and experimental case study, by estimating the joint-stiffness parameters of a given mechanism. The presented case study highlights the importance of a proper linearization equilibrium-point selection for a reliable and accurate identification of the joint parameters.

2. Equations of motion for a general multibody system

The governing equations of motion for a general multibody system are derived using the embedding technique [1], where the number of equations is equal to the number of the system's degrees of freedom. The equations of motion for a rigid multibody system can now be written as:

$$\delta \boldsymbol{q}^{\mathrm{T}} \left[\mathbf{M} \ddot{\boldsymbol{q}} - \boldsymbol{Q_e} \right] = 0 \,, \tag{1}$$

where $\mathbf{M}$ represents the mass matrix, $\boldsymbol{Q_e}$ the vector of the external forces acting on the system and $\boldsymbol{q}$ is the vector of the system coordinates, which consists of dependent $\boldsymbol{q}_{\mathrm{d}}$ and independent coordinates $\boldsymbol{q}_{\mathrm{i}}$. The virtual changes of the vector of the system coordinates can be expressed using only the virtual changes of the independent coordinates, as shown in [1]:

$$\delta \boldsymbol{q} = \begin{bmatrix} \delta \boldsymbol{q}_{\mathrm{d}} \\ \delta \boldsymbol{q}_{\mathrm{i}} \end{bmatrix} = \mathbf{B}_{\mathrm{i}} \delta \boldsymbol{q}_{\mathrm{i}} \,, \quad \mathbf{B}_{\mathrm{i}} = \begin{bmatrix} \mathbf{C}_{\mathrm{di}} \\ \mathbf{I} \end{bmatrix} . \tag{2}$$

Eq. (2) can now be inserted into Eq. (1) and rewritten only in terms of the independent coordinates:

$$\delta \boldsymbol{q}_{\mathrm{i}}^{\mathrm{T}} \mathbf{B}_{\mathrm{i}}^{\mathrm{T}} \left[\mathbf{M} \ddot{\boldsymbol{q}} - \boldsymbol{Q_e} \right] = 0 \,. \tag{3}$$

Since the components of the vector $\delta \boldsymbol{q}_{\mathrm{i}}$ are independent, their coefficients in Eq. (3) must be equal to zero and this leads to a system of differential equations:

$$\mathbf{B}_{\mathrm{i}}^{\mathrm{T}} \mathbf{M} \ddot{\boldsymbol{q}} - \mathbf{B}_{\mathrm{i}}^{\mathrm{T}} \boldsymbol{Q_e} = \mathbf{0} \,. \tag{4}$$

By writing the equations in terms of independent coordinates we obtain:

$$\overline{\mathbf{M}}_{\mathrm{i}} \ddot{\boldsymbol{q}}_{\mathrm{i}} = \overline{\boldsymbol{Q}}_{\mathrm{i}} \,, \tag{5}$$

where

$$\overline{\mathbf{M}}_{\mathrm{i}} = \mathbf{B}_{\mathrm{i}}^{\mathrm{T}} \mathbf{M} \mathbf{B}_{\mathrm{i}}, \quad \overline{\boldsymbol{Q}}_{\mathrm{i}} = \mathbf{B}_{\mathrm{i}}^{\mathrm{T}} \boldsymbol{Q_e} - \mathbf{B}_{\mathrm{i}}^{\mathrm{T}} \mathbf{M} \boldsymbol{\gamma}_{\mathrm{i}} \,, \quad \gamma_{\mathrm{i}} = \begin{bmatrix} \mathbf{C}_{\mathrm{d}} \\ 0 \end{bmatrix} . \tag{6}$$

Kinematic joints are key components in a multibody simulation. Usually joints are represented with idealized models which restrain the motion of the entire system by a set of kinematic constraints [1, 38]. This kind of formulation considers

the joints as perfect rigid elements and has the advantage for simple implementation and computational efficiency. However, physical phenomena such as clearance, misalignment, flexibility, friction or impact can highly influence the dynamic response of the joints and have a non negligible effect on the accuracy of the multibody model [38].
In our study the simple revolute joint between two links is considered. For two contacting bodies i and j the joint kinematic constraint can be written as:

$$\mathbf{R}^i + \mathbf{A}^i \overline{\mathbf{u}}_P^i = \mathbf{R}^j + \mathbf{A}^j \overline{\mathbf{u}}_P^j \,, \tag{7}$$

where $\mathbf{R}^i$ and $\mathbf{R}^j$ are the global position vectors of local body coordinate systems, $\overline{\mathbf{u}}_P^i$ and $\overline{\mathbf{u}}_P^j$ are the position vectors of the contacting point P defined with respect to the local coordinate systems and $\mathbf{A}^i$ and $\mathbf{A}^j$ are the planar transformation matrices.

3. Linearization and identification of the natural frequencies

The identification of natural frequencies by means of a numerical integration followed by the Fourier transform is a mathematically complex and time-consuming process. In contrast, this paper proposes an alternative method using the linearization approach as presented in [35]. The method relies on the linearization of the governing equation of motion at a specific equilibrium point. First, the Eq. (5) is rewritten by considering all of the dependencies:

$$\underbrace{\overline{\mathbf{M}}_{\mathrm{i}}(\boldsymbol{q}_{\mathrm{i}})\ddot{\boldsymbol{q}}_{\mathrm{i}} - \overline{\boldsymbol{Q}}_{\mathrm{i}}(\boldsymbol{q}_{\mathrm{i}}, \dot{\boldsymbol{q}}_{\mathrm{i}}, t)}_{\boldsymbol{h}} = \mathbf{0}. \tag{8}$$

The linearization of Eq. (8) is performed at the equilibrium point: $\boldsymbol{q}'_{\mathrm{i}} = (\ddot{\boldsymbol{q}}_{\mathrm{i},0}, \dot{\boldsymbol{q}}_{\mathrm{i},0}, \boldsymbol{q}_{\mathrm{i},0})$:

$$\delta \boldsymbol{h} = \left.\frac{\partial \boldsymbol{h}}{\partial \ddot{\boldsymbol{q}}_{\mathrm{i}}}\right|_{\boldsymbol{q}'_{\mathrm{i}}} \delta \ddot{\boldsymbol{q}}_{\mathrm{i}} + \left.\frac{\partial \boldsymbol{h}}{\partial \dot{\boldsymbol{q}}_{\mathrm{i}}}\right|_{\boldsymbol{q}'_{\mathrm{i}}} \delta \dot{\boldsymbol{q}}_{\mathrm{i}} + \left.\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{q}_{\mathrm{i}}}\right|_{\boldsymbol{q}'_{\mathrm{i}}} \delta \boldsymbol{q}_{\mathrm{i}} = \mathbf{0}, \tag{9}$$

where $\delta\ddot{\boldsymbol{q}}_{\mathrm{i}}$, $\delta\dot{\boldsymbol{q}}_{\mathrm{i}}$ and $\delta\boldsymbol{q}_{\mathrm{i}}$ are the variations about $\boldsymbol{q}'_{\mathrm{i}}$. Assuming Eq. (9) is homogeneous, it can be rewritten in the linear form as:

$$\overline{\mathbf{M}}_{\mathrm{i},0}\delta\ddot{\boldsymbol{q}}_{\mathrm{i}} + \overline{\mathbf{D}}_{\mathrm{i},0}\delta\dot{\boldsymbol{q}}_{\mathrm{i}} + \overline{\mathbf{K}}_{\mathrm{i},0}\delta\boldsymbol{q}_{\mathrm{i}} = \mathbf{0}, \tag{10}$$

where $\overline{\mathbf{M}}_{\mathrm{i},0}$ is the linearized mass matrix:

$$\overline{\mathbf{M}}_{\mathrm{i},0} = \left.\frac{\partial \boldsymbol{h}}{\partial \ddot{\boldsymbol{q}}_{\mathrm{i}}}\right|_{\boldsymbol{q}'_{\mathrm{i}}}, \tag{11}$$

$\overline{\mathbf{D}}_{\mathrm{i},0}$ is the linearized damping matrix:

$$\overline{\mathbf{D}}_{\mathrm{i},0} = \left.\frac{\partial \boldsymbol{h}}{\partial \dot{\boldsymbol{q}}_{\mathrm{i}}}\right|_{\boldsymbol{q}'_{\mathrm{i}}} \tag{12}$$

and $\overline{\mathbf{K}}_{\mathrm{i},0}$ is the linearized stiffness matrix:

$$\overline{\mathbf{K}}_{\mathrm{i},0} = \left.\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{q}_{\mathrm{i}}}\right|_{\boldsymbol{q}_{\mathrm{i}}'}. \tag{13}$$

The variables $\overline{\mathbf{M}}_{\mathrm{i},0}$, $\overline{\mathbf{D}}_{\mathrm{i},0}$ and $\overline{\mathbf{K}}_{\mathrm{i},0}$ are explicit functions of both the system parameters and the equilibrium points and yield linearizations about any equilibrium point.

The system's natural frequencies can be identified by neglecting the damping $\overline{\mathbf{D}}_{\mathrm{i},0}$ and solving the eigenvalue problem. First, Eq. (10) is rewritten without the damping:

$$\overline{\mathbf{M}}_{\mathrm{i},0}\delta\ddot{\boldsymbol{q}}_{\mathrm{i}} + \overline{\mathbf{K}}_{\mathrm{i},0}\delta\boldsymbol{q}_{\mathrm{i}} = \mathbf{0}. \tag{14}$$

In order to determine the eigenvalues, a solution is assumed in the form of:

$$\delta\boldsymbol{q}_{\mathrm{i}} = \boldsymbol{S}\mathrm{e}^{\omega t}, \tag{15}$$

where $\boldsymbol{S}$ is the vector amplitude and ω the circular frequency. Substituting Eq. (15) into Eq. (14), we obtain:

$$\left(\overline{\mathbf{K}}_{\mathrm{i},0} - \omega^2\overline{\mathbf{M}}_{\mathrm{i},0}\right)\boldsymbol{S} = \mathbf{0}. \tag{16}$$

Eq. (16) defines a generalized eigenvalue problem, which is then used to obtain the natural frequencies of the system.

4. Introduction of the system's equilibrium-point selection algorithm into the joint-parameter identification process

The joint-identification algorithm proposed in this article consists of four major steps that are schematically presented in Fig. 1. First, the equations for a given multibody system are derived using the embedding technique with a set of independent coordinates (Eq. (5)). For the purposes of the presentation a three-link mechanism with revolute joints is analyzed, where the stiffness in the joint is represented by rotational springs. The stiffness of the rotational springs will be treated as unknown joint parameters that are to be deduced using the developed parameter-estimation procedure. Nevertheless, it should be noted that the proposed parameter-identification procedure could be applicable to an arbitrary multibody system's parameters, such as masses, moments of inertia, damping, etc. however the focus of this research is limited to joint stiffness parameters.

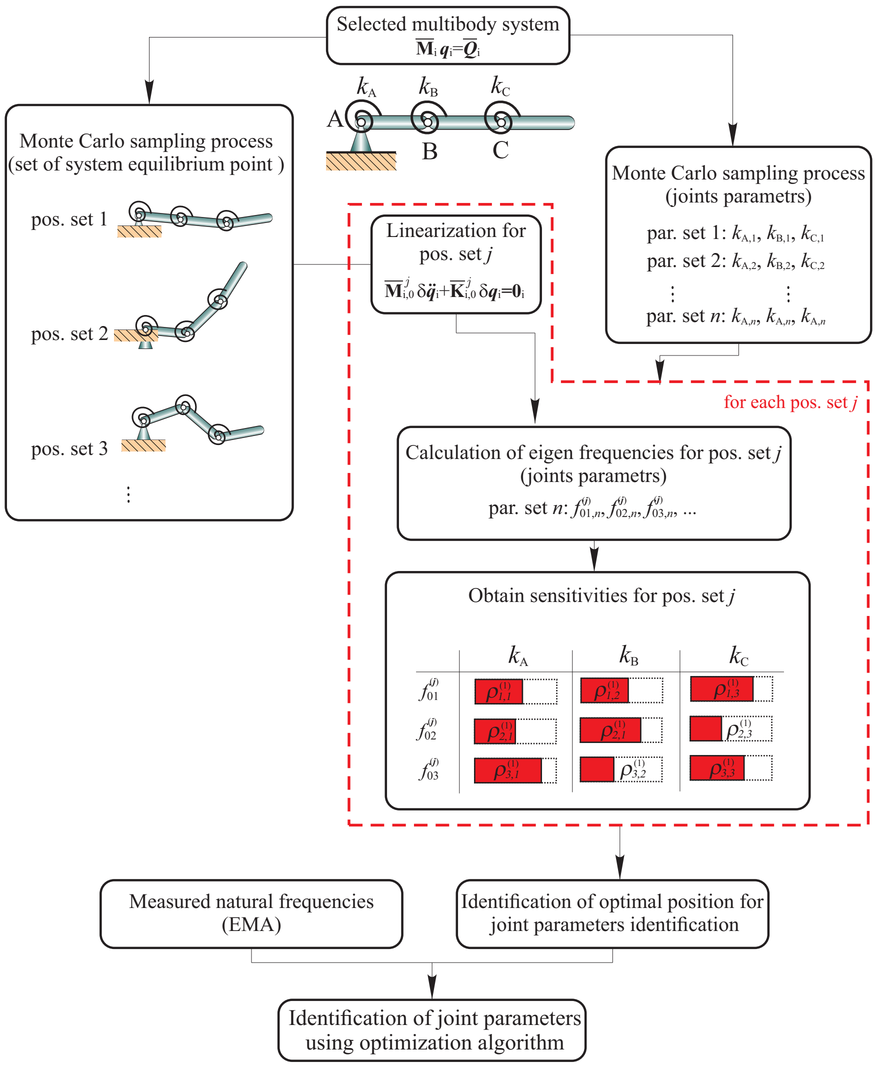


Figure 1: Schematic presentation of the multi-body parameter-identification algorithm.

In the second step the algorithm for the optimal system equilibrium-point selection is used. The algorithm is based on two successive Monte Carlo sampling processes that ensure the randomness of the generated data sets. For each equilibrium-point configuration j a linearization technique is applied and a set of system joint parameters using a Monte Carlo sampling process is constructed. For the given elementary case study (three-link mechanism) the matrix of the system's joint parameters can be written as:

$$\boldsymbol{\mathcal{K}} = \begin{bmatrix} k_{A,1} & k_{B,1} & k_{C,1} \\ k_{A,2} & k_{B,2} & k_{C,2} \\ \vdots & \vdots & \vdots \\ k_{A,n} & k_{B,n} & k_{C,n} \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{array}{l} \\ \\ \\ \leftarrow n\text{-th joint parameters set,} \\ \\ \end{array} \tag{17}$$

where the number of rows in the matrix corresponds to the size of the generated system's joint-parameter sets. Using the linearized equations of motion it is possible to deduce the system's natural frequencies for a given set of the system's joint parameters n. The obtained natural frequencies for the selected j-th system equilibrium point can be written in the matrix form as:

$$\boldsymbol{\mathcal{F}}^{(j)} = \begin{bmatrix} f_{01,1}^{(j)} & f_{02,1}^{(j)} & f_{03,1}^{(j)} & \cdots \\ f_{01,2}^{(j)} & f_{02,2}^{(j)} & f_{03,2}^{(j)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ f_{01,n}^{(j)} & f_{02,n}^{(j)} & f_{03,n}^{(j)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{18}$$

The sensitivity of the system's natural frequency versus a given joint-parameter set can now be obtained by calculating the Pearson's correlation coefficient [39]:

$$\rho_{i,m}^{(j)} = \frac{\mathrm{cov}(\boldsymbol{\mathcal{F}}_i^{(j)}, \boldsymbol{\mathcal{K}}_m)}{\sigma_{\boldsymbol{\mathcal{F}}_i^{(j)}} \sigma_{\boldsymbol{\mathcal{K}}_m}}, \quad j = 1, 2, 3, ..., \qquad m = 1, 2, 3, \tag{19}$$

where the $\boldsymbol{\mathcal{F}}_i^{(j)}$ represents the i-th column of the matrix $\boldsymbol{\mathcal{F}}^{(j)}$ and $\boldsymbol{\mathcal{K}}_m$ is the m-th column of the matrix $\boldsymbol{\mathcal{K}}$. The variables $\sigma_{\boldsymbol{\mathcal{F}}_i^{(j)}}$ and $\sigma_{\boldsymbol{\mathcal{K}}_m}$ are the standard deviation of the data in the vectors $\boldsymbol{\mathcal{F}}_i^{(j)}$ and $\boldsymbol{\mathcal{K}}_m$, respectively. The applied linearization technique enables efficient calculation of the natural frequencies and with this the sensitivities for large sets of equilibrium positions using Eq. (19). Thus, by inspecting all the equilibrium points and the obtained sensitivities, the optimal equilibrium point can now be deduced where the individual system's natural frequency is mainly influenced by only one joint parameter (Fig. 2).

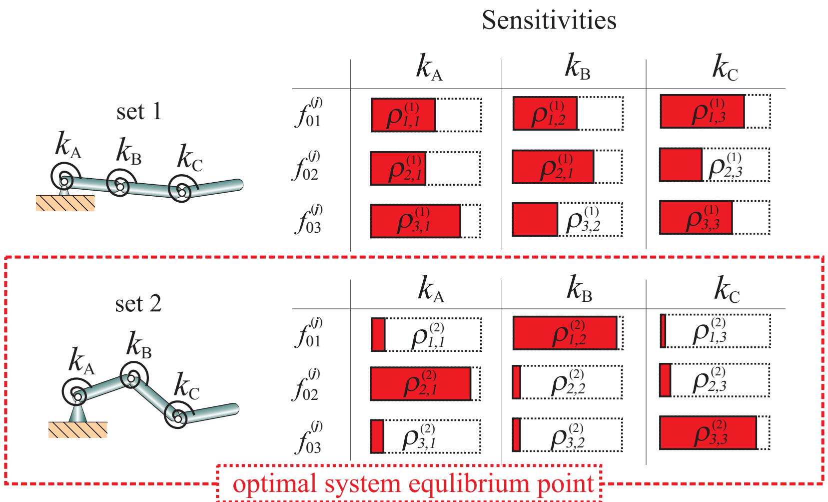


Figure 2: Identification of an optimal system equilibrium point for a multibody system.

In the third step the actual multibody system is positioned according to the chosen optimal equilibrium point and the experimental modal analysis (EMA) is performed. The results of the EMA are the natural frequencies and the corresponding mode shapes of the multibody system.
Finally, to deduce the unknown system's joint parameters an optimization process is performed by minimizing the discrepancy between the numerically and experimentally obtained system's natural frequencies:

$$\varepsilon = \sqrt{\sum_{i=1} (f_{i,exp.} - f_{i,num.})^2}, \tag{20}$$

where $f_{i,num.}$ is the i-th natural frequency obtained using the numerical model and $f_{i,exp.}$ the i-th experimentally obtained natural frequency. For solving the presented nonlinearly constrained optimization problem an interior point method is used [40]:

$$\begin{aligned} &\min \varepsilon \\ \text{subject to } &\boldsymbol{h}(x) = 0, \\ &\boldsymbol{g}(x) \leq 0, \end{aligned} \tag{21}$$

where ε represents the optimization objective function (Eq. (20)), $\boldsymbol{h}(x)$ the equality and $\boldsymbol{g}(x)$ the inequality of the constraint equations. In the given case study of a three-link mechanism the unknown system joint parameters are the torsional stiffnesses k_A, k_B and k_C.

5. Case study

The analyzed case study is a three-link mechanism, where the links are connected using revolute joints. The stiffness in the joints is modeled using a rotational spring, whereas the damping is neglected (Fig. 3). The system parameters are given in Table 1 where the superscripted index correlates to the given body in the system. It is assumed that the rotational stiffnesses in the joints are not affected by the relative rotation of the springs.

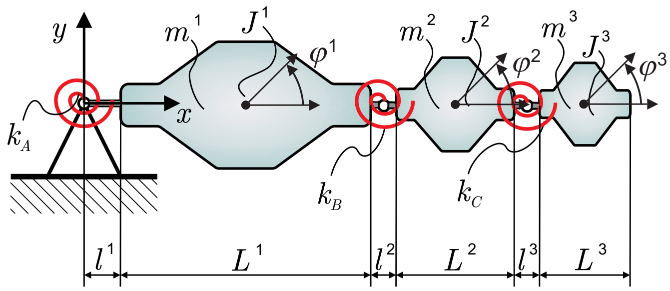


Figure 3: Multibody system under investigation.

The verification study of the proposed joint-parameter identification algorithm will first be performed on the numerical case study. An experimental case study will follow in order to demonstrate the dependency of the system's natural frequencies with respect to the joint parameters.

Table 1: The parameters of analyzed multibody system.

Symbol	**Value**
m^1; m^2; m^3 [g]	74.1; 54.3; 36.4
J^1; J^2; J^3 [kg mm^2]	15.8; 8.3; 3.8
l^1; l^2; l^3 [mm]	6.0; 6.0; 6.2
L^1; L^2; L^3 [mm]	60; 50; 40
k_A; k_B; k_C [$\frac{\mathrm{Nm}}{\mathrm{rad}}$]	21.1; 13.9; 5.8

5.1. Numerical verification

To demonstrate the efficiency of the proposed joint-parameter identification algorithm a numerical case study is performed. Although the system's joint parameters (e.g., the rotational stiffness) are known, it will be assumed that they represent unknown system parameters. Thus, by knowing the actual stiffness it will be possible to assess the accuracy of the proposed algorithm.

The essential step is to deduce the optimal system equilibrium point, on the basis of which the system's unknown joint parameters are deduced (Fig. 1). Therefore, a set of 1000 equilibrium points was established using a Monte Carlo simulation algorithm. In order to obtain the sensitivities (joint parameters → natural frequencies) a set of 1000 system joint parameters was also constructed. The natural frequencies were then obtained for a combined 1 million system configurations. The time required to calculate the natural frequencies for all of the system configurations was 80 seconds on a common laptop (i7 dual core processor, 24 Gb Ram). Based on the set of equilibrium points it was possible to deduce the system's configuration where the interplay between the different joint parameters for the given natural frequencies is minimized. In Fig. 4 two system configurations are presented to demonstrate the importance of proper system equilibrium-point selection during the joint-parameter identification.

Configuration 1 is the most common configuration, where the links are positioned in a straight line, whereas Configuration 2 represents the identified optimal position of the mechanism. For both configurations the calculated natural frequencies are presented in table 2 and will be considered as known variables for the system. In real applications the natural frequencies would normally be obtained using experimental modal analysis.

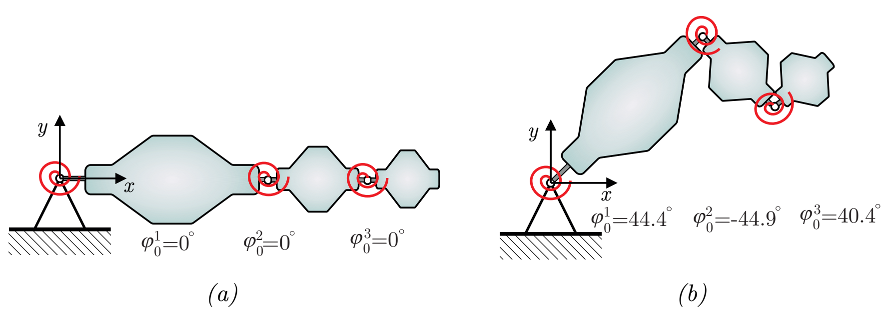
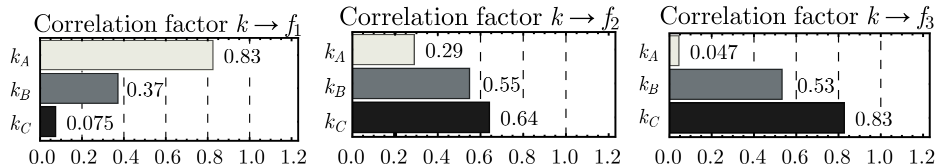


Figure 4: Analyzed configurations of the multibody system; a) Configuration 1, b) Configuration 2.

Table 2: The calculated natural frequencies for both analyzed configurations.

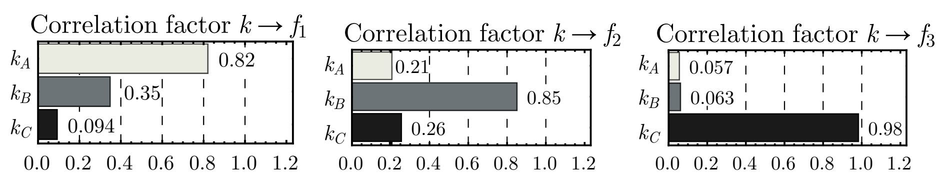
	Configuration 1	Configuration 2
f_1	19.9 Hz	22.7 Hz
f_2	83.1 Hz	58.5 Hz
f_3	229.8 Hz	101.2 Hz

By inspecting the correlation factors (Figs. 5, 6) it is evident that the correlation factors are practically the same for the first natural frequency. However, in the cases of the second and third natural frequencies a significant change in the correlation factors can be observed. Configuration 1 exhibits a strong interplay of the joint stiffness parameters for the system's natural frequencies: the individual natural frequency is significantly influenced by at least two joint stiffness parameters. However, in the case of Configuration 2 the individual natural frequency is influenced mainly by a single joint stiffness parameter.



(a) *(b)* *(c)*

Figure 5: Interplay between the joint stiffness coefficients for the system's natural frequencies for Configuration 1; a) First natural frequency f_1, b) Second natural frequency f_2, c) Third natural frequency f_3.



(a) *(b)* *(c)*

Figure 6: Interplay of joint-stiffness coefficients on the system's natural frequencies for Configuration 2; a) First natural frequency f_1, b) Second natural frequency f_2, c) Third natural frequency f_3.

The same conclusions regarding the influence of joint stiffness parameters on the system natural frequencies can be reached by examining the mode shapes for both configurations (Figs. 7, 8). Generally a higher joint sensitivity for a given natural frequency is exhibited as a higher deviation compared to the initial position. In the case of our three-link mechanism the deviation is seen in the form of an angle change between the initial link position and the mode shape link position. For Configuration 1 the first mode shape is mostly influenced by the first joint. The second and third mode shapes are sensitive for both the second and third joints but the influence of the third joint is more substantial for the third mode shape. In the case of Configuration 2 each mode shape mostly exhibits changes at only one joint while keeping the initial orientation at the other joints. This is in agreement with the correlations factors shown in Figure 6.

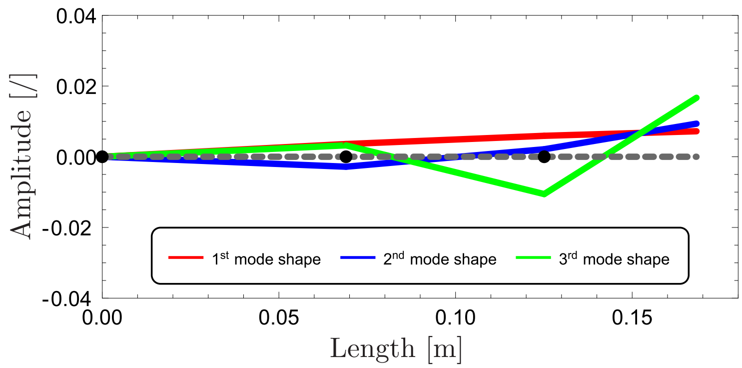


Figure 7: The first three mode shapes for Configuration 1.

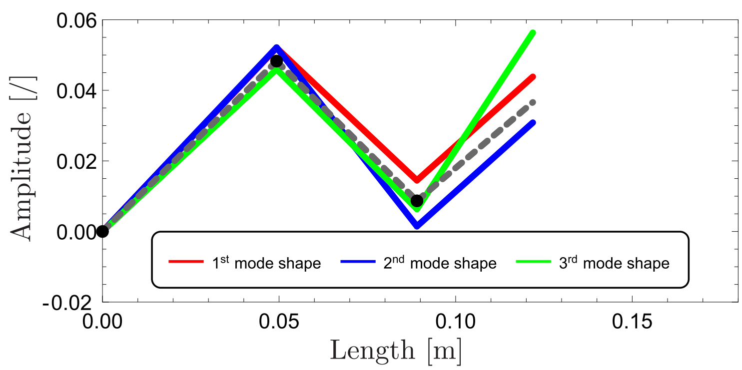


Figure 8: The first three mode shapes for Configuration 2.

By identifying the optimal equilibrium configurations it is now possible to deduce the system's joint parameters by using the optimization approach (Eq. (21)). Table 3 lists the calculated stiffness parameters for both equilibrium configurations. As can be seen, the stiffness parameters that were deduced from Configuration 2 are far more accurate than the ones obtained using Configuration 1. Configuration 2 can therefore be considered more appropriate for performing the optimization process.

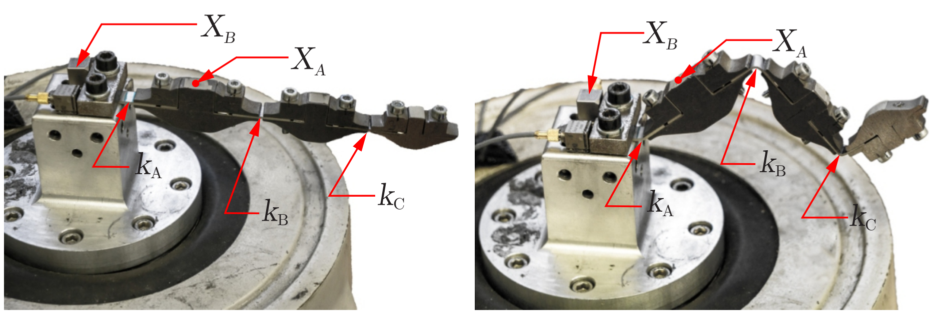
The presented case study highlights the importance of a proper linearization equilibrium-point selection and confirms that the proposed algorithm in this paper is reliable for the multibody system parameter identification.

Table 3: *Results of the optimization process.*

			k_A	k_B	k_C
Reference values	Value	$\left[\frac{\mathrm{Nm}}{\mathrm{rad}}\right]$	21.10	13.90	5.77
Starting estimations	Value	$\left[\frac{\mathrm{Nm}}{\mathrm{rad}}\right]$	30.00	20.00	10.00
Configuration 1	Value	$\left[\frac{\mathrm{Nm}}{\mathrm{rad}}\right]$	20.08	17.59	4.79
	Error [%]		-4.83	26.54	-16.97
Configuration 2	Value	$\left[\frac{\mathrm{Nm}}{\mathrm{rad}}\right]$	21.2	13.6	5.82
	Error [%]		0.5	-2.00	1.00

5.2. Experimental case study

Here, the actual experimental setup is used to demonstrate the effect of the system's joint parameters on its natural frequencies. The system is analyzed in both selected equilibrium points, as shown in Fig. 9. The joint stiffness was represented by thin metal sheets of different thickness. The dimensions of the metal sheets were set to obtain approximately the same joint stiffness as those presented in Table 1.



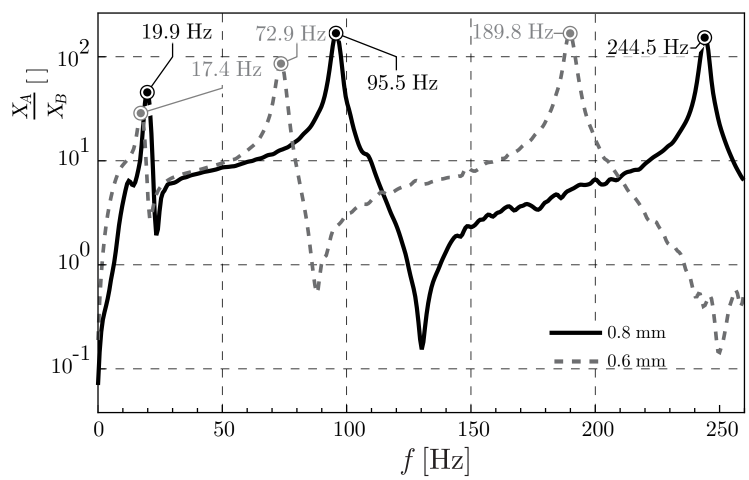
(a) *(b)*

Figure 9: Experimental set-up used in the vibration test; (a) Configuration 1, (b) Configuration 2.

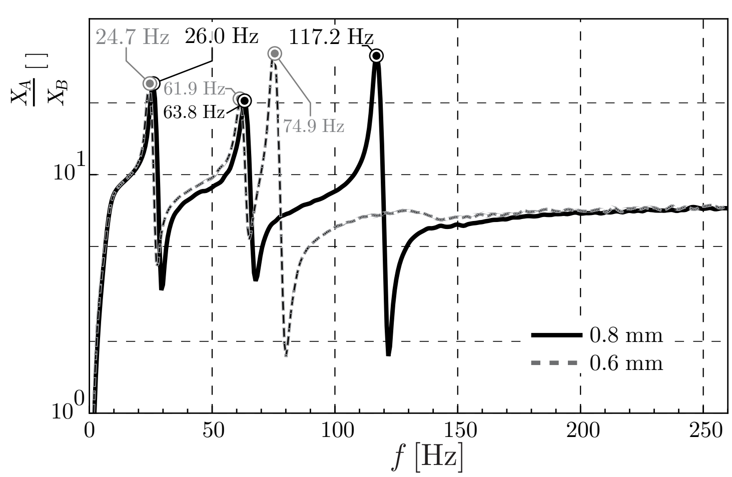
The system's natural frequencies were deduced based on the frequency-response functions (FRFs) measured between the points X_A/X_B. The system was excited with a random vibration profile generated by an electrodynamic shaker. Based on the obtained FRFs it was possible to deduce the system's natural frequencies, as shown in Fig. 10.
In order to demonstrate the effect of the joint parameters on the system's natural frequencies, the stiffness of the third joint was altered by decreasing the thickness of the metal sheet from 0.8 mm to 0.6 mm. By analyzing the system

in Configuration 1 it is evident that a change of the stiffness in the third joint influenced the second joint as well as the third natural frequency (Fig. 10). By inspecting the system in Configuration 2 it is clear that the stiffness in joint 3 mainly affected the third natural frequency and had practically no influence on the other two natural frequencies.

Thus, it is demonstrated that the proposed system-parameter-identification algorithm can be used to identify the parameters of real multibody systems and is not limited just to ideal numerical case studies.



(a)



(b)

Figure 10: Frequency-response functions of the analyzed multibody system; (a) Position 1, (b) Position 2.

6. Conclusion

A parameter-identification procedure for multibody dynamic systems has been developed and its feasibility verified on a numerical case study and on a real structure. The procedure identifies the unknown model parameters of a multibody system by minimizing the discrepancy between the numerically obtained and measured system's natural frequencies. The natural frequencies of a real structure are usually easy to obtain requiring only minimal measuring equipment and since the proposed method uses only discrete eigenfrequency values it is also robust with regards to the measurement noise. The formulation employs the embedding technique to express the multibody system's equations followed by a linearization procedure to efficiently deduce the system's natural frequencies. In general the input for our algorithm are the linearized equations of motion. These can be obtained either by using the linearization procedure proposed in our paper or by the alternative approach to rewrite the equations of motion in the linear form with respect to selected parameters. The performance of the parameter-estimation algorithm was substantially improved by incorporating the algorithm for an optimal linearization equilibrium-point selection. The numerical case-study example highlights the accuracy of the method and with this the importance of a proper equilibrium-point selection. As the optimization algorithm minimizes the cost function it is desirable that an individual stiffness coefficient has a dominant influence on only one natural frequency. Here, a three-link mechanism was analyzed and it was shown that the equilibrium-point selection has a major influence on the accuracy of the identified system's parameters. Moreover, a real case study is presented to validate the numerical model as well as to show that parameter localization to a particular natural frequency is not only limited to numerical case studies, but is also inherent to real case systems. Although only stiffnes joint parameters were identified in the presented case study the proposed study could also be used to find other joint parameters, e.g the rigid body model could be upgraded to also include damping.

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